Leading the Next

IEEE 802.1AS
Timing and Synchronization and its applications

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Outline

- Audio Video Bridging IEEE 802.1 projects
- AVB and Wireless Requirements
- Relation of IEEE 802.1AS and IEEE 1588 V2
- Network examples
- Messages
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- Summary
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  - Appendix I – End-to-end application reference model example
  - Appendix II – Definition of MTIE
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Audio Video Bridging

- Audio/Video Bridging (AVB) refers to a set of standards being developed in IEEE 802.1 to allow the transport of time-sensitive traffic over bridged local area networks (LANs)
  - One goal is to allow a single network infrastructure to carry both time-sensitive and non-time-sensitive traffic
- AVB applications will include
  - Digital video
  - High-Fidelity digital audio
  - Gaming
  - Traditional data traffic (non-time-sensitive)
- Major use is expected to be in the residence, but also intended for AVB applications in enterprises
- Two key AVB requirements
  - Guaranteed QoS
    - Resource reservation and admission control for media streams
    - Meet jitter, wander, time synchronization and latency requirements of applications
    - Along with end-to-end requirements, must consider reference model(s) to determine allocation to AVB network
  - Minimal (or no) administration required by users
    - Provisioning should not be required on an ongoing basis

AVB Standards in IEEE 802.1

802.1AS – network synchronization

- Timing and synchronization for time-sensitive applications in bridged local area networks
- PAR approved by NesCom; Draft D0.3.3 available [2] (many sections still incomplete)
- can be used to transport synchronization in other networks besides AVB networks

802.1Qat – bandwidth reservation and admission control

- PAR approved by NesCom; initial (incomplete) draft available

Bridge forwarding and queueing behavior

- PAR being developed; has been submitted by 802.1 Chair to EC

Guidelines/"Best Practices"

- PAR being developed

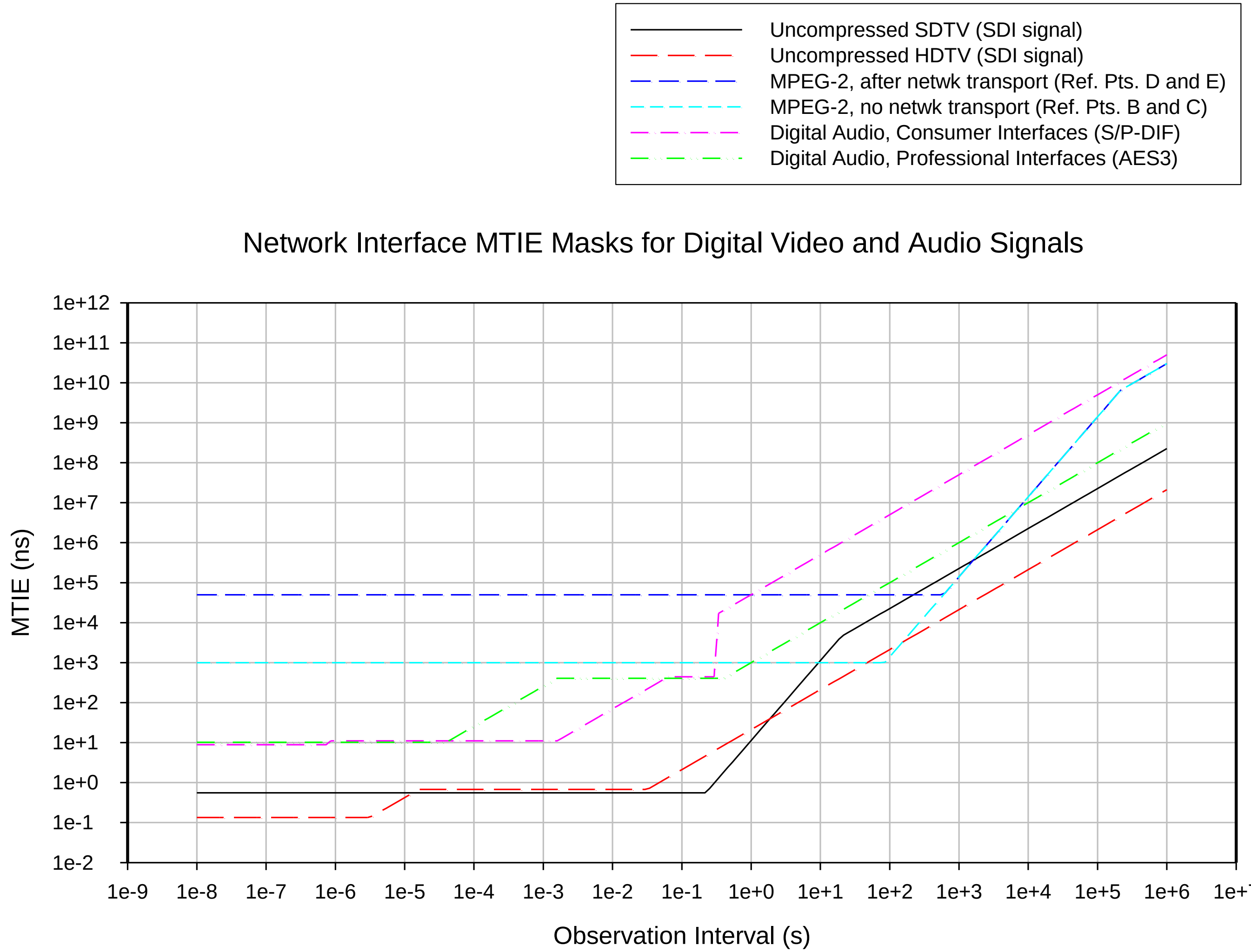
AVB End-to-End Jitter and wander requirements

(see[1] and references given there)

Requirement	Uncompressed SDTV	Uncompressed HDTV	MPEG-2, with network transport	MPEG-2, no network transport	Digital audio, consumer interface	Digital audio, professional interface
Wide-band jitter (UIpp)	0.2	1.0	50 μs peak-to-peak phase variation requirement (no measurement filter specified)	1 μs peak-to-peak phase variation requirement (no measurement filter specified)	0.25	0.25
Wide-band jitter meas filt (Hz)	10	10			200	8000
High-band jitter (UIpp)	0.2	0.2			0.2	No requirement
High-band jitter meas filt (kHz)	1	100			400 (approx)	No requirement
Frequency offset (ppm)	±2.79365 (NTSC) ±0.225549 (PAL)	±10	±30	±30	±50 (Level 1) ±1000 (Level 2)	±1 (Grade 1) ±10 (Grade 2)
Frequency drift rate (ppm/s)	0.027937 (NTSC) 0.0225549 (PAL)	No requirement	0.000278	0.000278	No requirement	No requirement

AVB End-to-End MTIE requirements

End-to-End Application Jitter and Wander Requirements Expressed as MTIE Masks [1] (see Appendix II for MTIE definition)



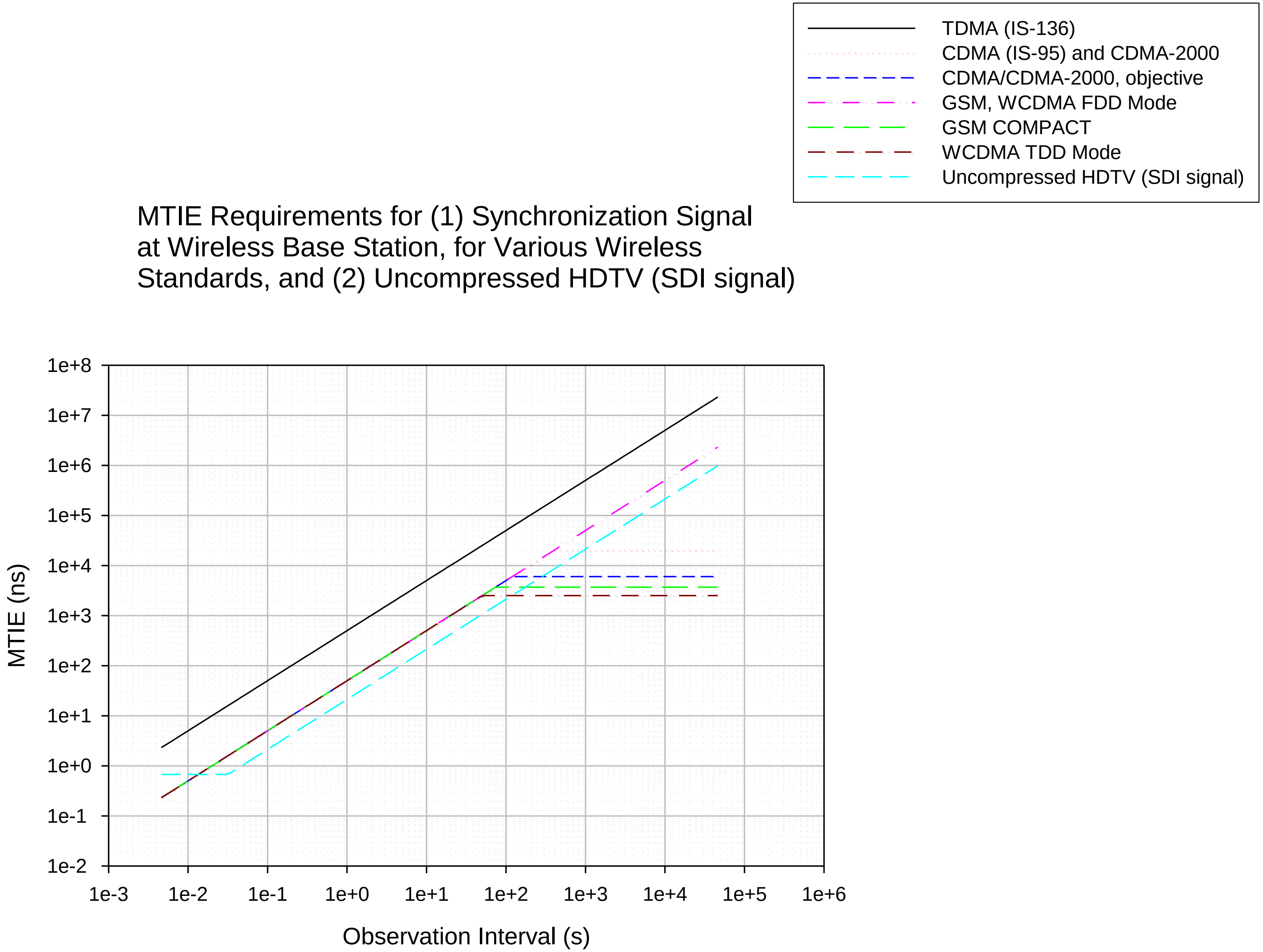
AVB Time Synchronization Requirements

- In general, a multi-media stream may contain multiple audio and/or video streams, possibly transported to different locations, e.g.
  - Multiple audio tracks from the same program transported to speakers in different locations
  - The same audio track transported to multiple speakers simultaneously
  - Voice and corresponding video streams from the same program being played simultaneously (lip-synch)
  - Video animation with accompanying audio
- Reference [9] describes the results of experiments that investigated the maximum skews that could be tolerated for various types of related streams before degradation in QoS would be perceived
  - Lip-synch; Video animation with accompanying audio
    - $\pm$80 ms
  - Tightly coupled audio and images
    - $\pm$5 ms
  - Tightly coupled audio (e.g., audio streams delivered to multiple speakers)
    - $\pm$10 μs

Base Station Timing Requirements

Requirement	TDMA	CDMA	GSM	CDMA2000	WCDMA FDD	WCDMA TDD
Maximum Frequency Offset (ppm)	0.5	0.05	0.05	0.05	0.05	0.05
Maximum Phase Offset (μs)	No requirement	± 10 (relative to UTC)	48/13 (GSM COMPACT; relative to other base stations)	± 10 (relative to UTC)	No requirement	2.5 (relative to other base stations)
Desired Maximum Phase Offset (not strict requirement) (μs)	No objective	± 3 (relative to UTC)	No objective	± 3 (relative to UTC)	No objective	No objective
Maximum Observation Interval for Phase Offset (s)	Not Applicable	28800 (8 hr)	Not specified	28800 (8 hr)	Not Applicable	Not Applicable

Base Station MTIE Requirements



IEEE 1588 V2 - the basis for IEEE 802.1AS

- IEEE 1588 Precision Time Protocol (PTP) V2 allows several different network architectures
  - Chains of boundary clocks (BCs), possibly with ordinary clocks (OCs) at the endpoints
  - Ordinary and/or boundary clocks connected through end-to-end (E2E) transparent clocks (TCs)
  - Ordinary and/or boundary clocks connected through peer-to-peer (P2P) TCs
  - IEEE 1588 V2 does not support direct connection of P2P and E2E TCs
    - Interworking of E2E and P2P TCs must occur through a BC
- IEEE 1588 V2 (like V1) does not contain specifications that guarantee performance (jitter, wander, time synchronization)
- IEEE 1588 V2 allows the definition of profiles for specific applications
  - Profile may specify whether certain optional features of IEEE 1588 (e.g., fault tolerance, security enhancements) are used for that application
  - Profile may define ranges and default values for various parameters (e.g., clock or port parameters)
  - Profile may limit the architectural options that the application may use (e.g., profile may specify that an application shall use only BCs, OCs, and P2P TCs)
- IEEE 1588 V2 (like V1) may be used over a variety of communication technologies
  - IEEE 1588 V2 does not address synchronization over wireless networks

IEEE 802.1AS profile of IEEE 1588 V2

- Subset of IEEE 1588 V2 to transport synchronization over wired Ethernet (full-duplex, 802.3 links)
  - Specifies which 1588 V2 features and architectural options will be used
    - E.g., clocks, messages, optional features, etc.
  - Specify ranges and default values for relevant parameters
- Specify additional requirements to ensure acceptable end-to-end performance for applications
  - E.g., node clock requirements, endpoint PLL filter requirements
- Contains additional specifications for transport of synchronization over 802.11 wireless networks
  - IEEE 802.11v messages and facilities will be used to transport synchronization over 802.11 links
  - Synchronization will be transported between 802.3 and 802.11 networks through boundary clock
  - Wired and wireless networks will use single Best Master Clock Algorithm (BMCA) for Grandmaster (GM) selection; entire 802.1AS network forms a single PTP domain
  - See [3] for details of Sync transport in wireless networks (here the focus is on wired networks)

IEEE 1588v2 clock types and AVB use

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❑Ordinary clock (OC)

- Has a single port
- Can be master or slave; an AVB network will have 1 master (termed the Grandmaster (GM))
- Primary purpose is to provide synchronized time at an endpoint; in AVB also used at each bridge

❑Boundary Clock (BC)

- Has multiple ports
- At most one port is in slave state; remaining ports are normally in master or passive state
- Primary purpose is to transfer timing at a network node
- Specialized 802.1AS BC used within 802.1AS network, e.g.,
  - Interface between wired and wireless 802.1AS network (wireless AP)
  - Interface to networks that are not 802 but sufficiently similar that they can use 802.1AS (e.g., MOCA, IEEE 1394)

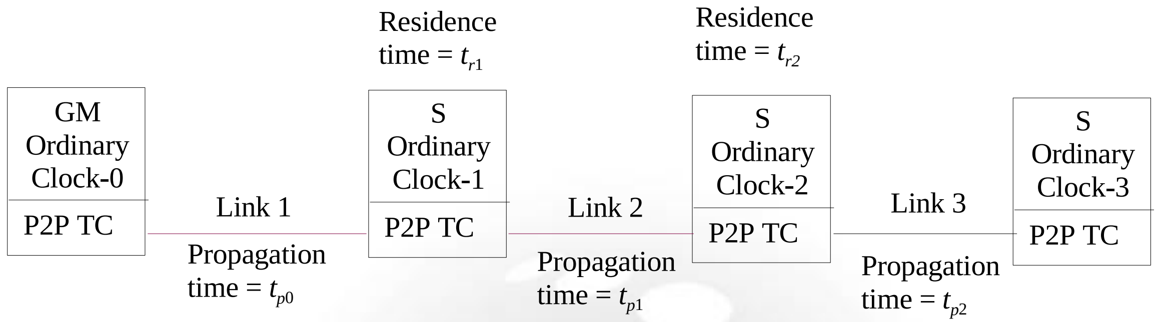
❑Peer-to-Peer Transparent Clock (P2P TC)

- Not part of master/slave hierarchy, but can syntonize to Grandmaster
- Primary purposes are to (1) measure residence time of synchronization-related messages that traverse a node to correct for variable delay through the node, and (2) measure propagation delays between itself and adjacent P2P TCs, BCs, or OCs

❑End-to-End Transparent Clock (E2E TC)

- Same as P2P TC, but does not measure propagation delays between itself and adjacent clocks
- Not used in AVB

Synchronizing Clocks through P2P TCs (1)



- Each Sync message may spend an indeterminate amount of time in each node (e.g., due to queueing for the outgoing link and other resources) before transmission to the next node
  - This time is termed the *residence time*
  - Even if the Sync message has priority, the priority will likely not be preemptive
- Residence time at each node is accumulated in a correction field of the Sync or Follow_Up message
  - Follow-up is used if the P2P TC is not capable of making an accurate residence time measurement and placing it in the Sync message on-the-fly (this would require hardware assist)

Synchronizing Clocks through P2P TCs(2)

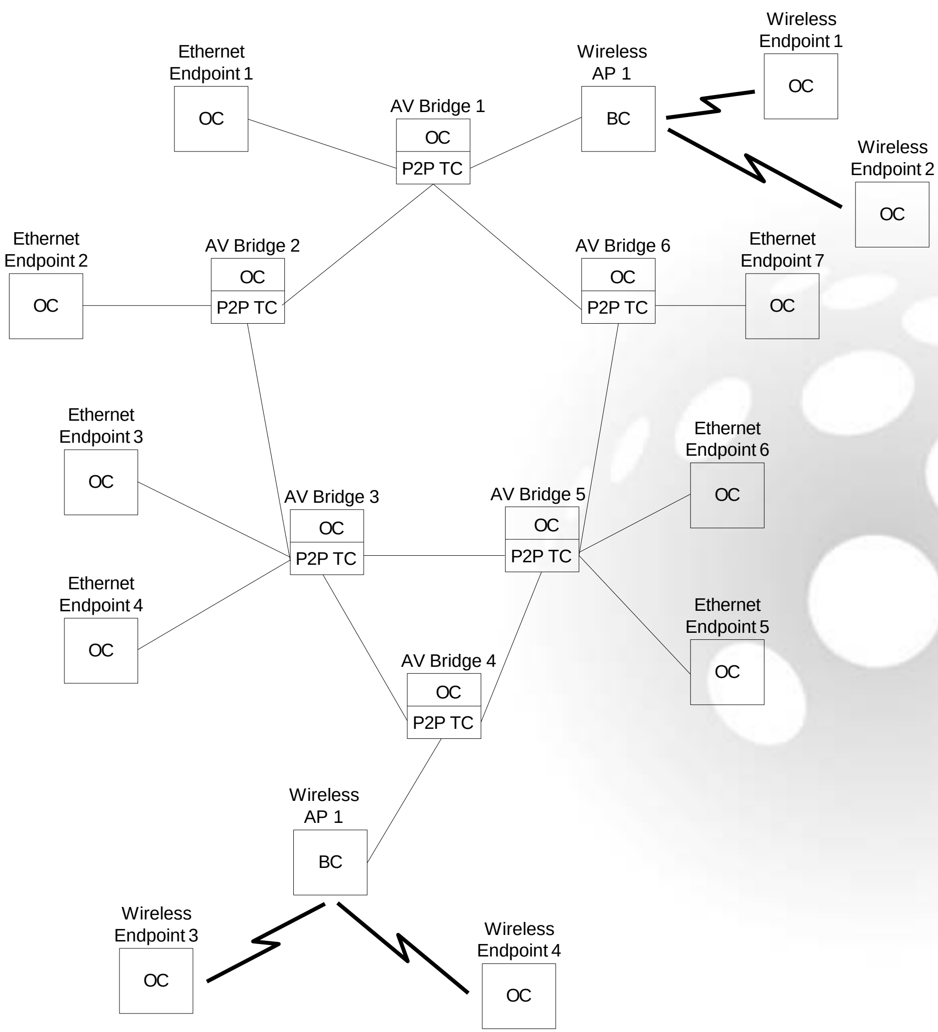
- P2P TCs also measure propagation times relative to their immediate neighbors
  - Avoid having to send Delay_Req and Delay_Resp messages
  - Avoids scaling issues when a GM has many slaves
  - Is beneficial when there is a network reconfiguration and a resulting new GM
    - Propagation delays are measured on every link and immediately available, while the Delay_Req/Delay_Resp mechanism only measures propagation delays on the paths between the GM and slaves
- The slave offset computation is

$$slave\_offset = t_2 - t_1 - total\_propagation\_plus\_residence\_time$$

$$total\_propagation\_plus\_residence\_time = \sum_{i=1}^{N-1} t_{ri} + \sum_{i=1}^{N} t_{pi}$$

$$Sync\_correction\_field + Follow\_Up\_correction\_field = \sum_{i=1}^{N-1} \left(t_{ri} + t_{pi} \right)$$

PTP Devices used in 802.1AS Network



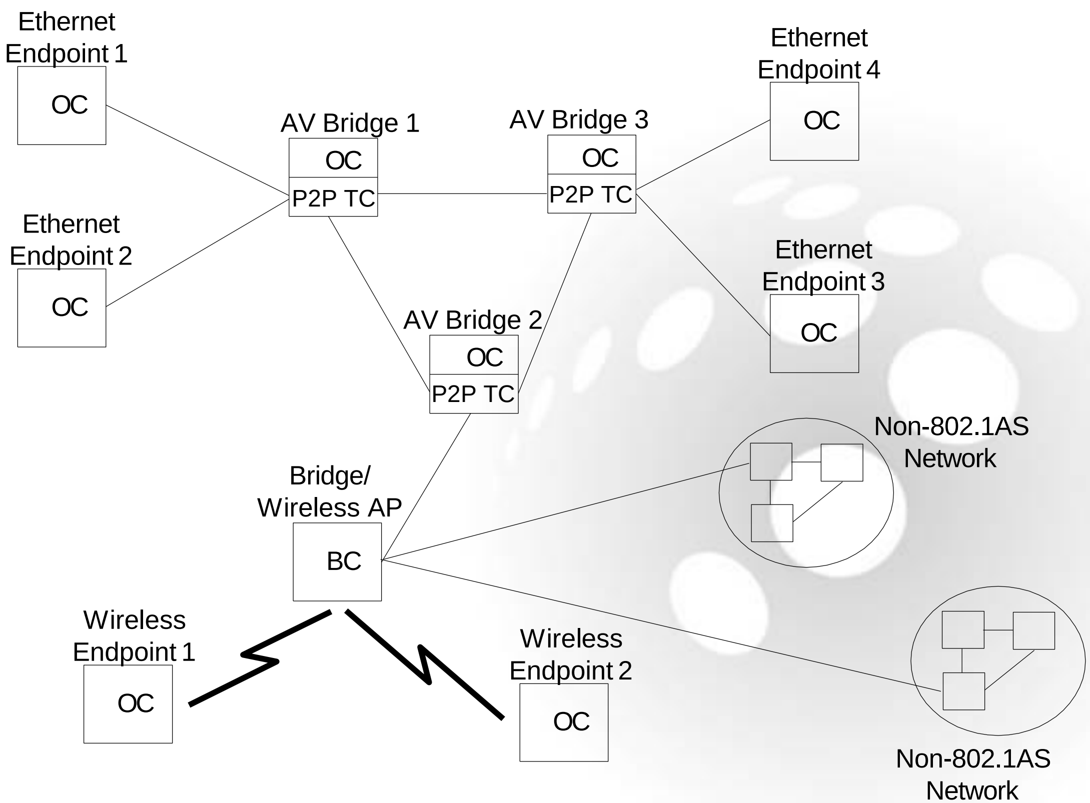
OC	Ordinary Clock
BC	Boundary Clock
P2P TC	Peer-to_Peer Transparent Clock

- A general 802.1AS network does require a P2P TC function to be contained in an AV Bridge

- A general 802.1AS network in principle need not require an OC function to be contained in an AV Bridge. In practice an OC function will be present, as the added cost of it is minimal

- A general 802.1AS network does require a BC function at the interface between an 802.3 and 802.11 network (a BC is needed to separate different network technologies, as various PTP parameters (e.g., Sync interval) may be different)

Connecting to non-802.1AS Networks



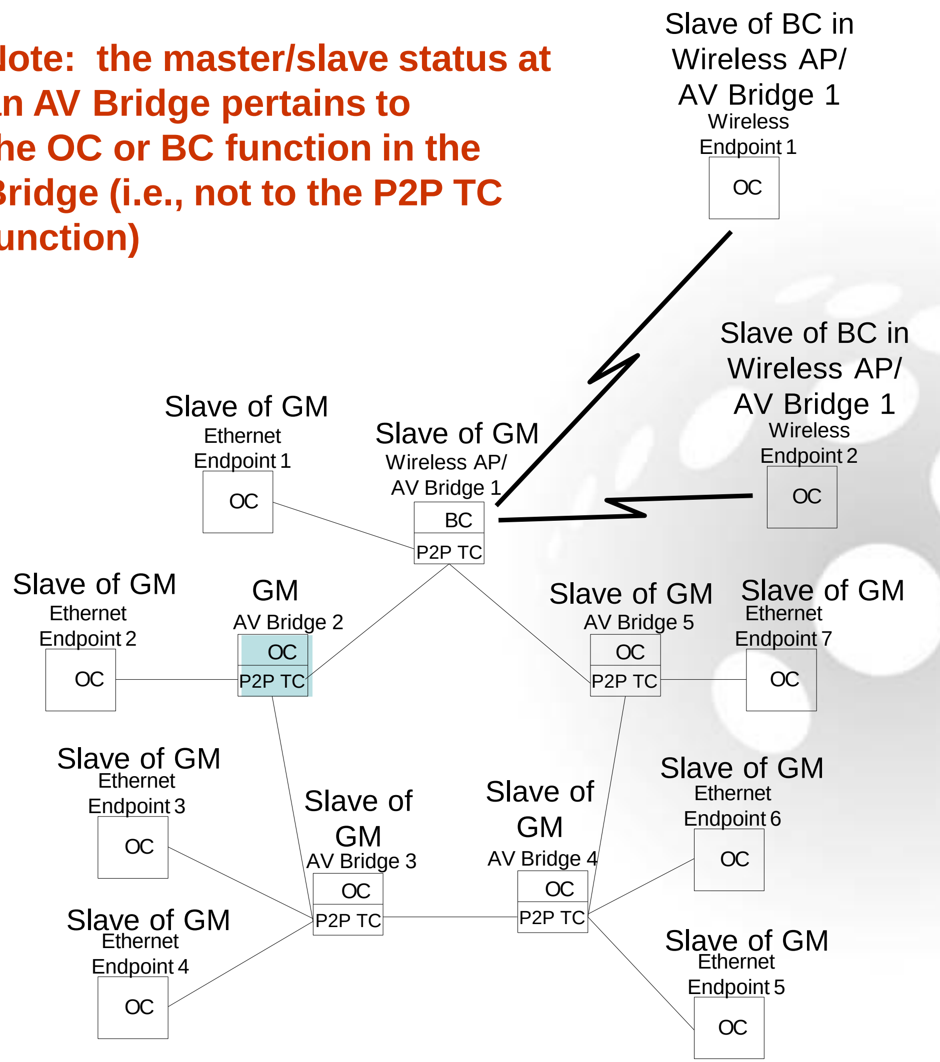
OC	Ordinary Clock
BC	Boundary Clock
P2P TC	Peer-to_Peer Transparent Clock

– An 802.1AS network requires a BC at the interface to a non–802.1AS network

–The non–802.1AS network might be a PTP network that meets IEEE 1588 V2 but not the particular profile specified in 802.1AS

Example Master/Slave Hierarchy in 802.1AS

Note: the master/slave status at an AV Bridge pertains to the OC or BC function in the Bridge (i.e., not to the P2P TC function)



OC	Ordinary Clock
BC	Boundary Clock
P2P TC	Peer-to_Peer Transparent Clock

- Entire 802.1AS network forms a single PTP domain
- Synchronization hierarchy is determined by BMCA
- An OC can be master or slave
- A BC port can be master or slave; a BC has at most one port in the PTP_Slave state
- A P2P TC is neither master nor slave
- All devices except P2P TCs process Announce messages
- All devices except P2P TCs are capable of issuing Announce messages (and do so when in the PTP_Master state)

802.1AS Messages

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- PTP message used in 802.3 and 802.11 portions
  - Announce (general message)
- PTP messages used in 802.3 portions
  - Synch (event message)
  - Follow_Up (general message)
  - Pdelay_Req (event message)
  - Pdelay_Resp (event message)
  - Pdelay_Resp_Follow_Up (general message)
- 802.11v messages used in 802.11 portions (see [3] for details)
  - Presence Request (message is timestamped)
  - Presence Response (message is not timestamped)
  - ACK (message is timestamped)

802.1AS Clock Requirements

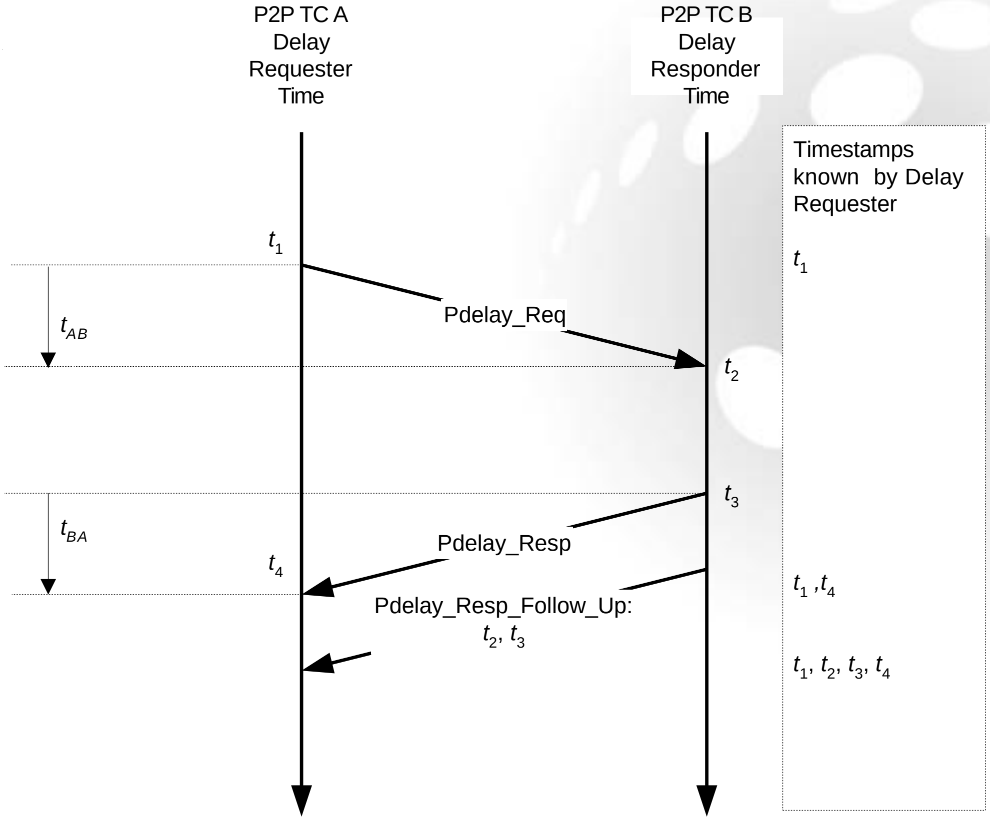
- All clocks will be required to have ±100 ppm free-run accuracy
- Not decided yet if there will be a requirement on noise generation and frequency stability for AVB applications
  - If there is such a requirement, it will be consistent with the requirement of low cost (e.g., oscillator cost << $1.00) for for consumer electronics applications
- For OCs, there will be a single clock quality
  - Likely PTP Class 4 (formerly PTP Stratum)
  - PTP Clock Identifier will reflect current status of clock, and will be set automatically
    - Clock Identifier indicates local oscillator time source, i.e., source via means other than PTP
      - e.g., atomic clock, GPS, NTP, DFLT
      - DFLT used for clocks of Class 3 or greater for which the time source is not atomic, GPS, NTP, nor set by hand or by a management procedure of unspecified accuracy
      - see 1588 V2 for details

BMCA in 802.1AS

- User will be able to set Priority1 field (externally settable absolute priority)
- User will not set Priority2 field (externally settable priority considered after Priority1, Class, Clock Identifier, Variance, and whether one clock is a BC)
- Best master selection in 802.1AS network will not be influenced by Class, Clock Identifier, Priority2, or Variance
  - The extent to which an 802.1AS OC can execute a simplified BMCA yet preserve interoperability with non-802.1AS PTP networks through BCs is being investigated
    - May assign default values to Priority2 and Variance

Measurement of Prop Time on an 802.3 Link

- Propagation times are measured using the Pdelay mechanism
- The mechanism is executed independently in both directions (the figure below (adapted from [7]) shows only one direction to simplify the presentation)



$$t_{AB} = t_2 - t_1$$

$$t_{BA} = t_4 - t_3$$

$$t_{mean-prop} = \frac{t_{AB} + t_{BA}}{2}$$

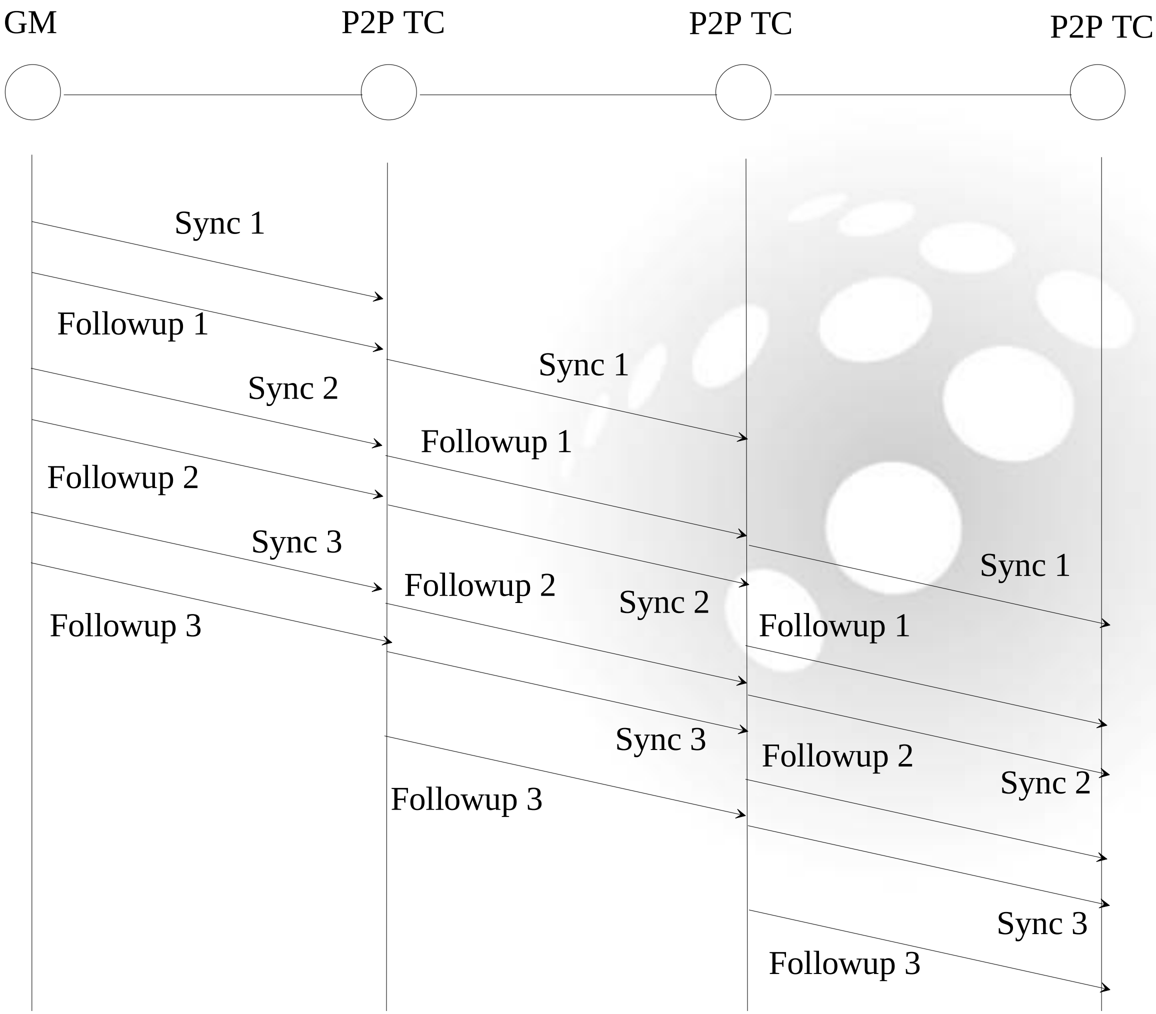
$t_{mean-prop}$ is the propagation time if the propagation times in the two directions are the same

Holding of Sync messages

- In current Draft, both on-the-fly and follow-up operation are allowed
  - To provide for interoperability 802.1AS specifies that Sync messages should be held until a corresponding Follow_Up message arrives
  - .Is allowed but not required by 1588

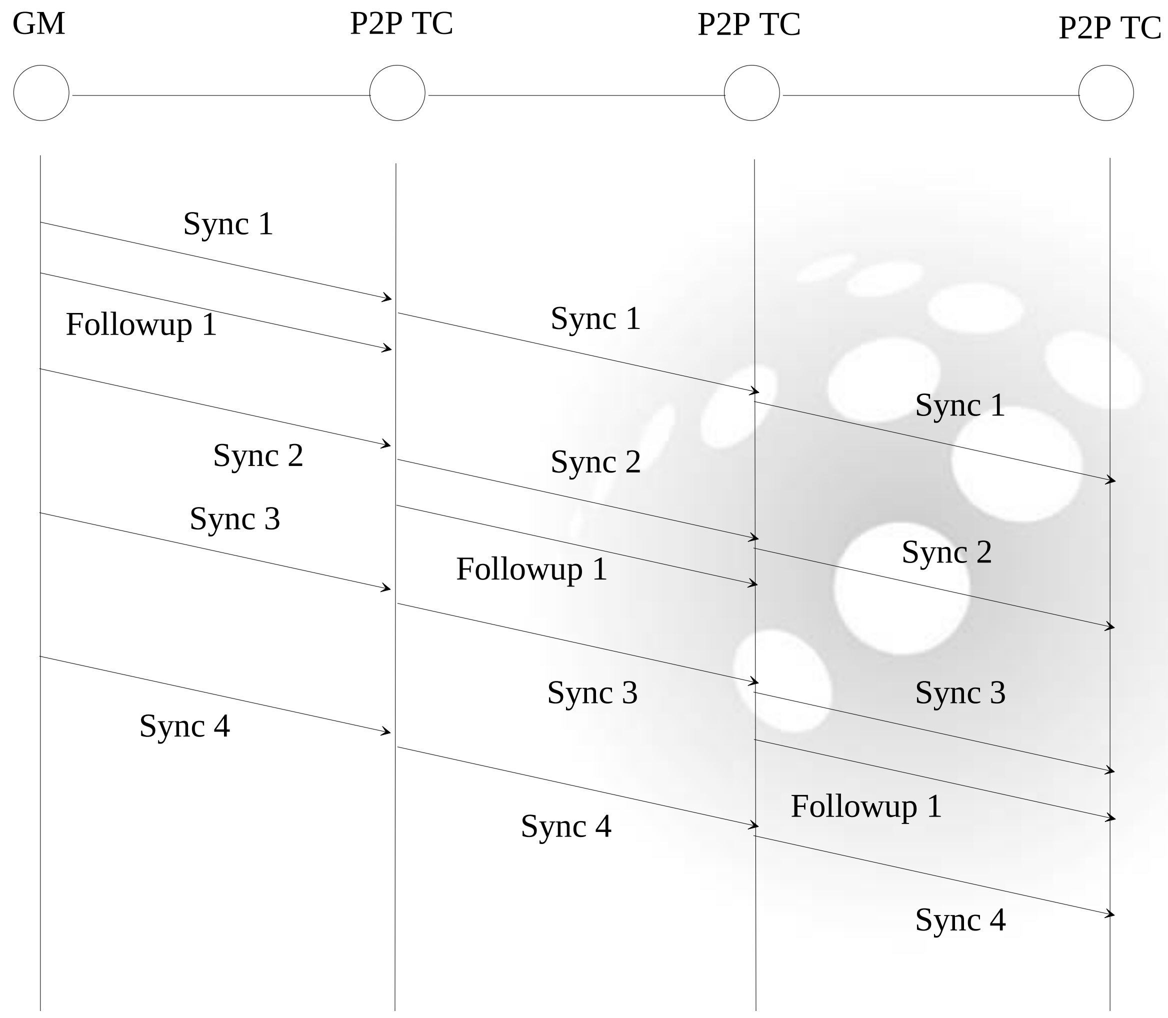
- If Master or P2P TC is follow-up it will set the PTP_ASSIST bit of Sync message to 1
- An on-the-fly P2P TC does the following if the PTP_ASSIST bit is set to 1:
  - The Sync message is held until the Follow_Up message arrives (and its measured residence time reflects this)
  - The Follow_Up message is transmitted unaltered
- A follow-up TC does the following if the PTP_ASSIST bit is set to 1:
  - The Sync message is held until the corresponding Follow_Up message arrives
  - The content of the correction field of the Follow_Up message is added to the correction field of the Sync message
  - The sum of the measured residence time for the Sync message and the propagation delay for the link on which the Sync message arrived is placed in the correction field of a new Follow_Up message

Synchronization through P2P TCs



- ❑Example of timing of transport of Sync and Follow_Up messages through a network, illustrating the holding of a Sync message until the corresponding Follow_Up message arrives

Synchronization through P2P TCs



Note: The Followup messages corresponding to Sync2 and Sync3 are not shown to keep the diagram from being too cluttered.

- Example of timing of transport of Sync and Follow_Up messages through a network, illustrating the accumulation of multiple Sync messages at P2P TCs if Follow_Up messages are not held
- This accumulation would occur in AVB networks if Sync messages are not held due to much longer processing time for Follow_Up than Sync in low-end processors

Syntonizing a P2P TC to the Grandmaster

- P2P TC will be required to meet specified requirements (e.g., frequency accuracy, etc.)
  - Can meet the requirement by syntonizing (I.e., synchronizing in frequency) the P2P TC to the GM
  - However, 802.1AS allows the requirement to be met in any manner the designer chooses (e.g., could provide a sufficiently high quality oscillator)
- If the P2P TC is syntonized, it is done as follows:
  - The frequency offset of the GM relative to the P2P TC is measured every M^{th} Sync message (at present, $M = 10$)
  - For every M^{th} Sync message, estimate the GM time when the Sync message is received
  - The P2P TC clock time when the Sync message arrives is timestamped
  - The frequency offset of the GM relative to the P2P TC is measured by computing the elapsed P2P TC clock time and GM clock time for the interval of M Sync messages
  - The P2P TC frequency is adjusted
    - May be implemented in hardware or software

Endpoint Filtering

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- All stringent filtering (e.g., using phase-locked loops (PLLs)) will be performed at endpoints
  - This will allow the expense of the filtering to be associated with end applications that require it
  - Different applications have different jitter, wander, and time synchronization requirements
    - E.g., uncompressed digital video requirements are most stringent, followed by high-fidelity digital audio, followed by compressed (e.g., MPEG-2, MPEG-4) digital video
- Filter requirement will be expressed using a transfer mask, with specified equivalent 3dB bandwidth, gain peaking, roll-off, and frequency range over which the requirement applies

Endpoint Filtering

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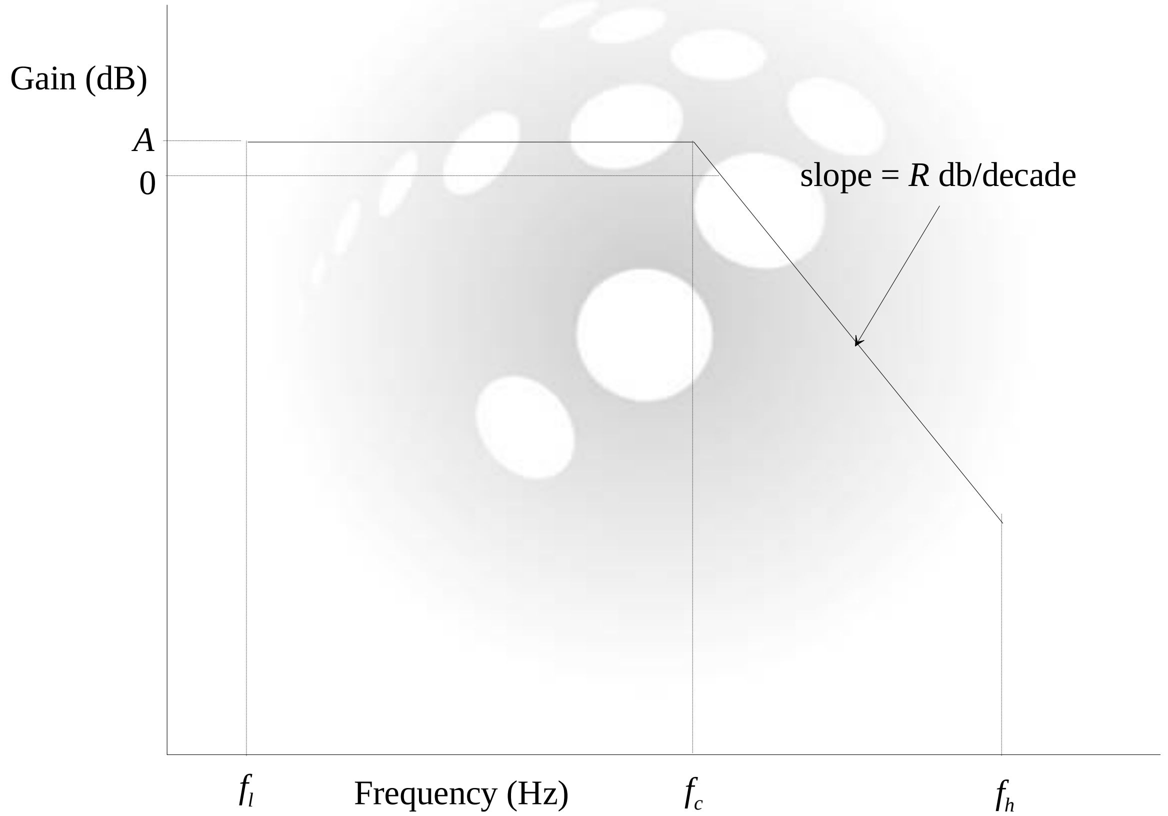
Illustration of transfer requirement mask for endpoint filter

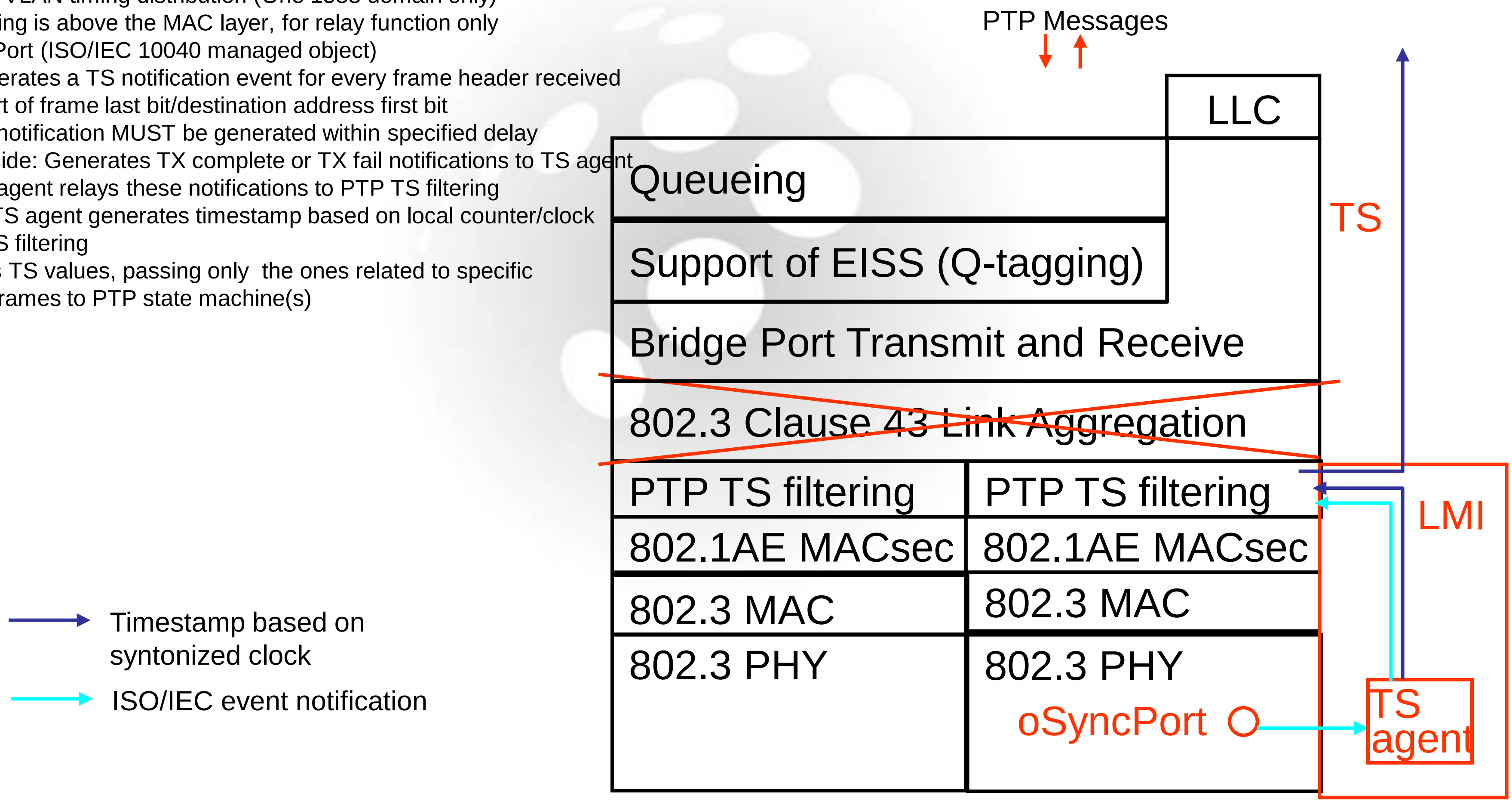


802.1AS Architecture for AV Bridge with Wired Interfaces

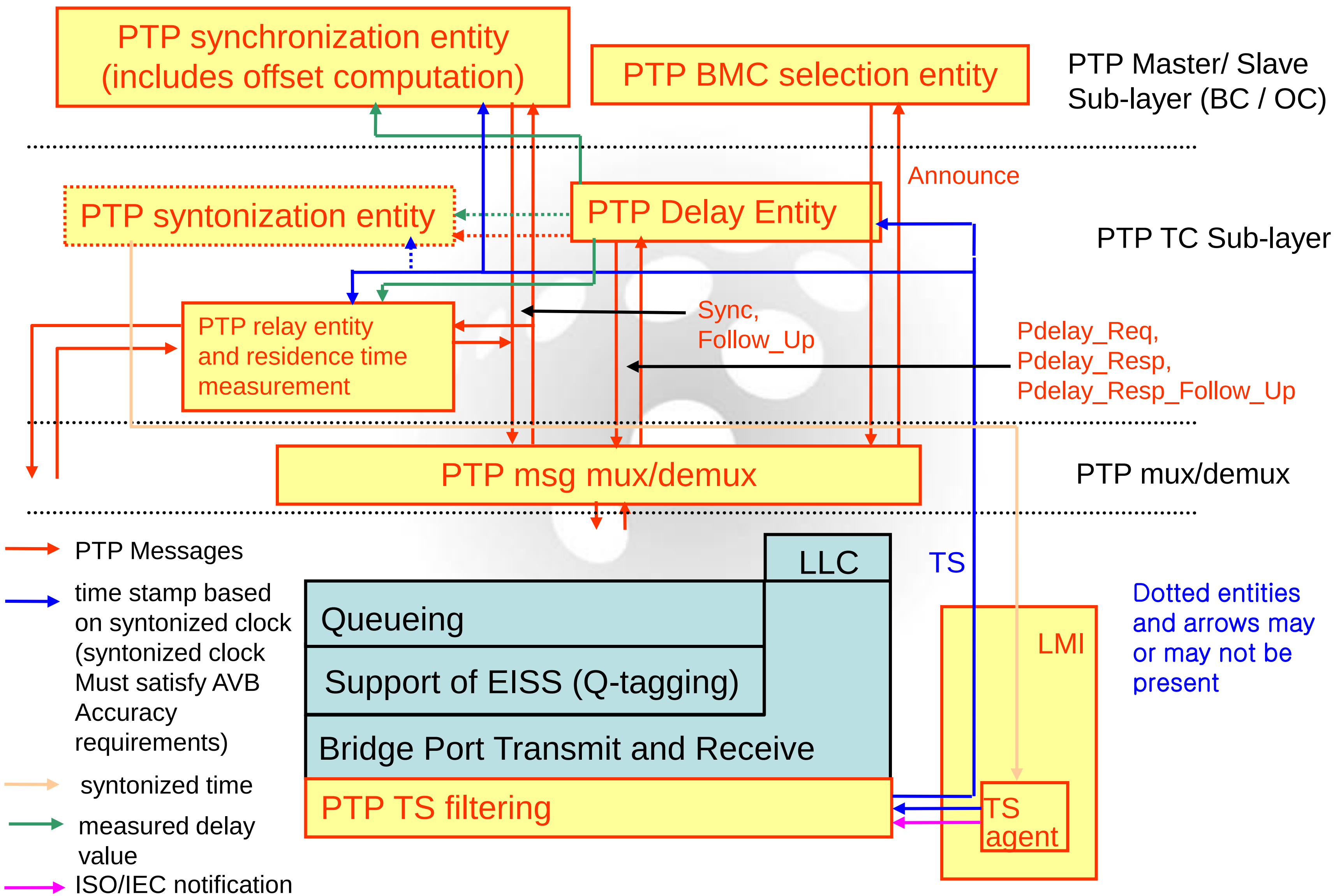
Current 802.1/802.3 layering based on “baggy pants” model
Figure and text below is taken from [4], with some modifications
Additional modifications are possible (work is in progress)

Notes:

1- No link aggregation for now
2- No per VLAN timing distribution (One 1588 domain only)
3- Queueing is above the MAC layer, for relay function only
4- oSyncPort (ISO/IEC 10040 managed object)
  a- Generates a TS notification event for every frame header received
    - Start of frame last bit/destination address first bit
    - TS notification MUST be generated within specified delay
  b- TX side: Generates TX complete or TX fail notifications to TS agent
    - TS agent relays these notifications to PTP TS filtering
5- Local TS agent generates timestamp based on local counter/clock
6- PTP TS filtering
  o Filters TS values, passing only the ones related to specific PTP frames to PTP state machine(s)



802.1AS Architecture for AV Bridge with Wired Interfaces – PTP MAC Client



Simulation Cases

❑Parameters common to all simulation cases

- Free-run clock accuracy = ± 100 ppm
  - Initialize frequency offset of each clock randomly within this range
- Phase measurement granularity = 40 ns (25 MHz free-running clocks)
- Frequency measurement granularity = 2.3283 x 10^{-10} (32 bit accuracy)
- Consider up to 10 hops (AVB will require 7 hops; we exceed that to see how much MTIE increases beyond 7 hops)
- Sync interval = 10 ms
- Frequency update interval = 100 ms
- Pdelay turnaround time (t3 – t2 on slide 10) = 1 ms
- Asymmetry in PHY latency and cable delay not modeled
- Free running clock has phase noise modeled as in [5]
- Endpoint filter is 2^{nd} order, with 0.1 dB gain peaking
- Three cases are simulated with 3dB bandwidth of: 1 Hz, 0.1Hz,.0.01Hz

Case 1 Simulation Results

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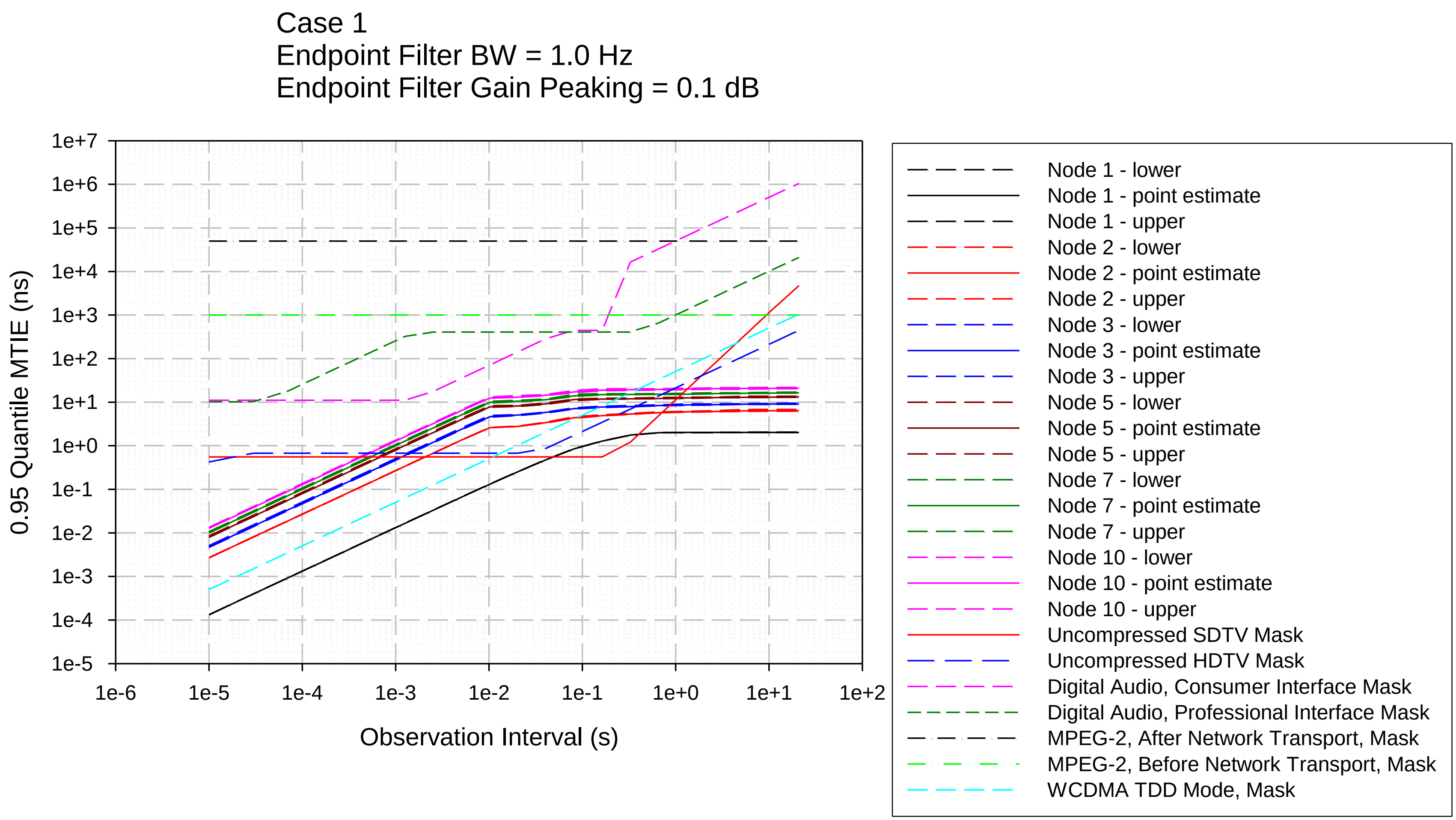
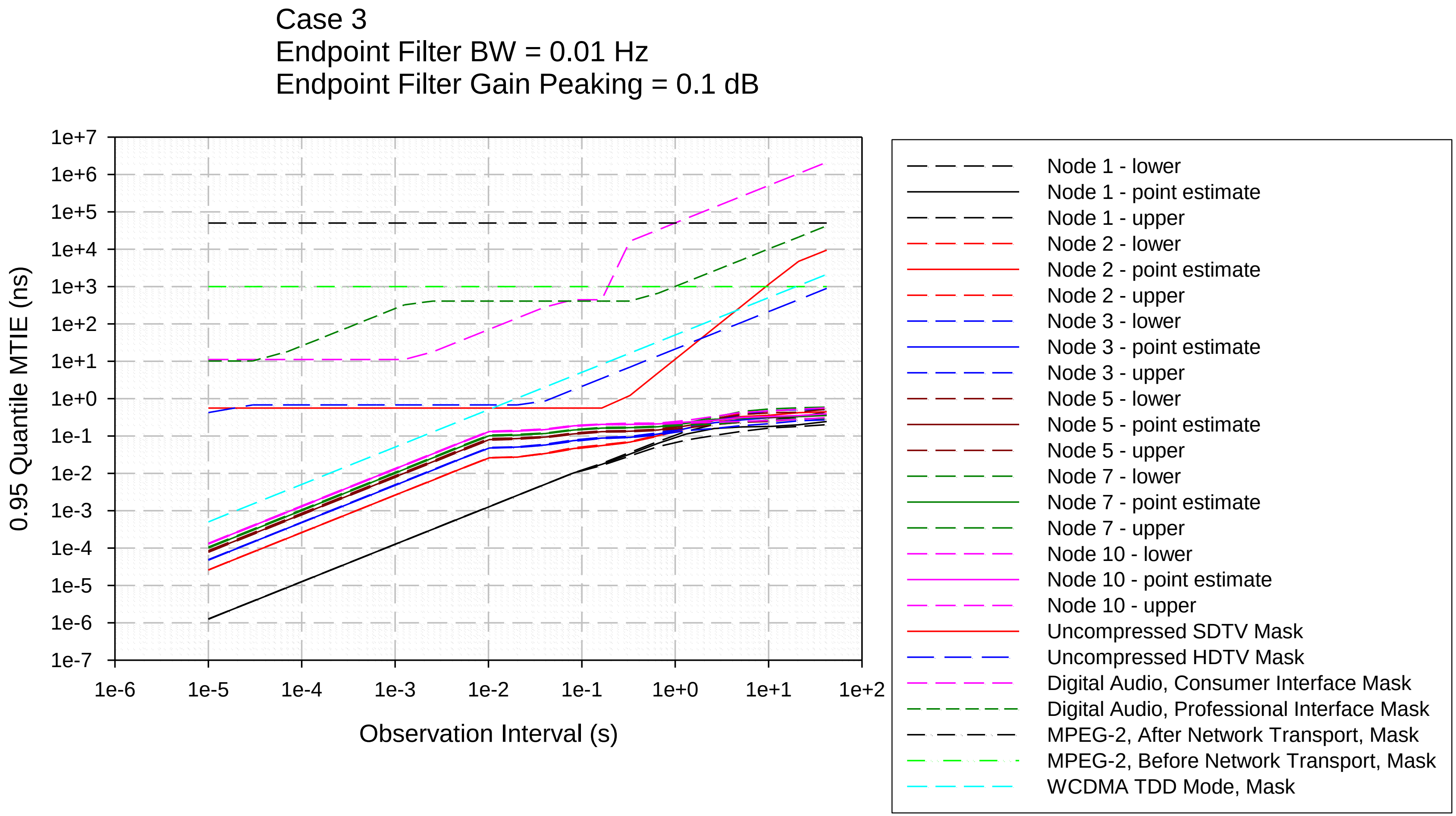


Case 3 Simulation Results



Discussion of Results

- Meeting the requirements for uncompressed digital video requires a filter bandwidth not much more than 0.01 Hz
- Digital audio and compressed digital video requirements are met with 1 Hz filter
  - The amount of margin present indicates may be able to use a bandwidth between 1 and 10 Hz
  - Note that asymmetry in PHY latency is not modeled; more analysis is needed to determine PHY requirements
- Compressed digital video (i.e., MPEG-2, MPEG-4) can likely be met with bandwidth wider than 10 Hz
- Examination of the computed residence times and propagation time errors indicated that the main contribution to phase error is the effect of the 40 ns phase measurement granularity
  - The results for a single hop are much better than those for multiple hops because, for a single hop, residence time is not used
  - Effect is due to truncation of residence time measurement to the next lower multiple of 40 ns

Base Station Synchronization

- Primary source of time synchronization in CDMA and WiMAX is GPS
  - GPS receivers that are optimized for timing can achieve accuracy in the order of 100 ns over several hours
- GPS reception does suffer from radio interference
  - To allow for extended periods (hours) of interference requires an expensive clock with good holdover performance or a secondary network timing reference signal with time-frequency information
  - Good holdover performance comes at a price
- Indoor BS locations usually have no GPS reception

- Desirable to have on option to provide Base Station synchronization via an Ethernet backhaul network

Achievable Accuracy with 802.1AS

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Accuracy achievable in normal operation

- Time accuracy has not yet been simulated, but will be no worse than 40 ns per hop due to worst case phase measurement granularity, and could be as good as 4 ns per hop with 250 MHz clock
- In worst case for AVB, there may be a contribution of up to 100 ns per hop due to asymmetry in PHY latency and cable delay
  - For fiber transport, there is no cable delay asymmetry
  - For integrated PHY with calibration (e.g., in the prototype being developed) the PHY latency asymmetry would be smaller
- For 7 hops, time accuracy should be no worse than 1 μs; much better performance is feasible

Accuracy during network reconfiguration

- Determined by Spanning Tree reconfiguration time
- Phase transient will be less than 300 ns for 2 s reconfiguration time

Summary

- IEEE 802.1AS is being developed by the 802.1 AVB TG
  - Will use a subset of IEEE 1588 V2, and add
    - Specification of various 1588 parameters and which options are used (i.e., specify 1588 profile)
    - Guarantee end-to-end application performance
    - Transport synchronization over 802.11 wireless network portions
- Transport synchronization in wired network portion using peer-to-peer transparent clock
- Interface between wired and wireless network portions and between 802.1AS and non-802.1AS networks contains a boundary clock
- Initial simulation results show that end-to-end performance requirements for Audio/Video applications in a residence can be met
- 802.1 AS can potentially be used for synchronization of Base Stations through Ethernet backhaul networks

References

1. Geoffrey M. Garner, *End-to-End Jitter and Wander Requirements for ResE Applications*, Samsung presentation at May, 2005 IEEE 802.3 ResE SG meeting, Austin, TX, May 16, 2005. Available via http://www.ieee802.org/3/re_study/public/index.html.
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3. Kevin Stanton, *Clock Synchronization over 802.11 for Home A/V Applications*, 2006 Conference on IEEE 1588, Gaithersburg, MD, USA, October 2 – 4, 2006.
4. *Time Synchronization and 802 Models*, contributors include Dirceu Cavendish, George Claseman, Geoffrey Garner, Franz-Josef Goetz, and Kevin Stanton, presentation for IEEE 802.1 AVB wireless group conference calls, available at http://www.ieee802.org/1/files/public/docs2006/as-cavendish-802ModelforTS-060911.pdf.
5. Geoffrey M. Garner and Kees den Hollander, *Analysis of Clock Synchronization Approaches for Residential Ethernet*, 2005 Conference on IEEE 1588, Winterthur, Switzerland, October 10 – 12, 2005.

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6. ITU-T Recommendation G.810, *Definitions and Terminology for Synchronization Networks*, ITU-T, Geneva, August, 1996, Corrigendum 1, November, 2001.
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8. Athanasiois Papoulis, *Probability, Random Variables, and Stochastic Processes*, Third Edition, McGraw-Hill, 1991, pp. 254 – 255.
9. Ralf Steinmetz, *Human Perception of Jitter and Media Synchronization*, IEEE JSAC, Vol. 14, No. 1, January, 1996.

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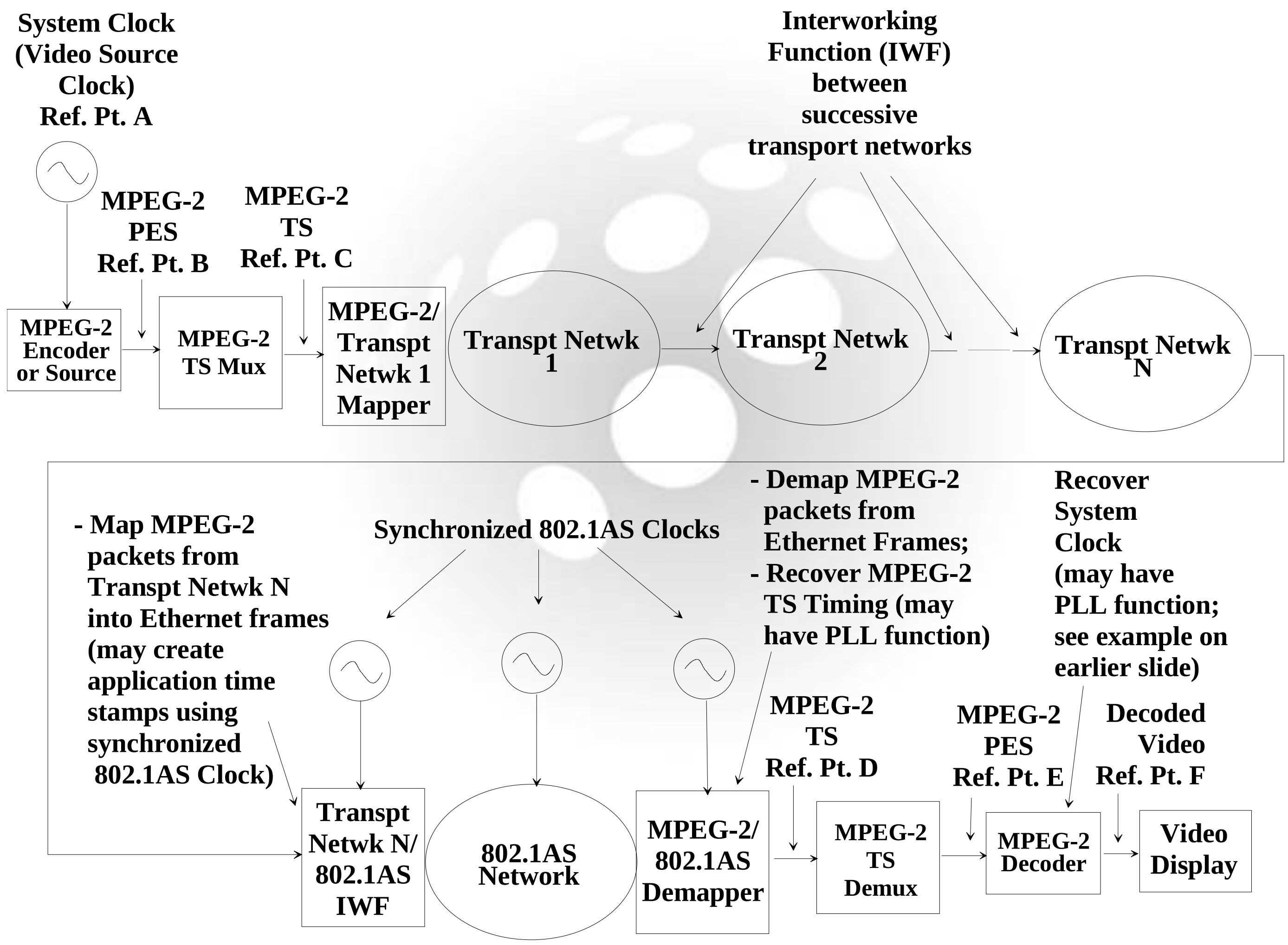
Example Reference Model for Transport of MPEG-2 Video over Service Provider Networks and 802.1AS Network [1]



Appendix II - Definition of MTIE

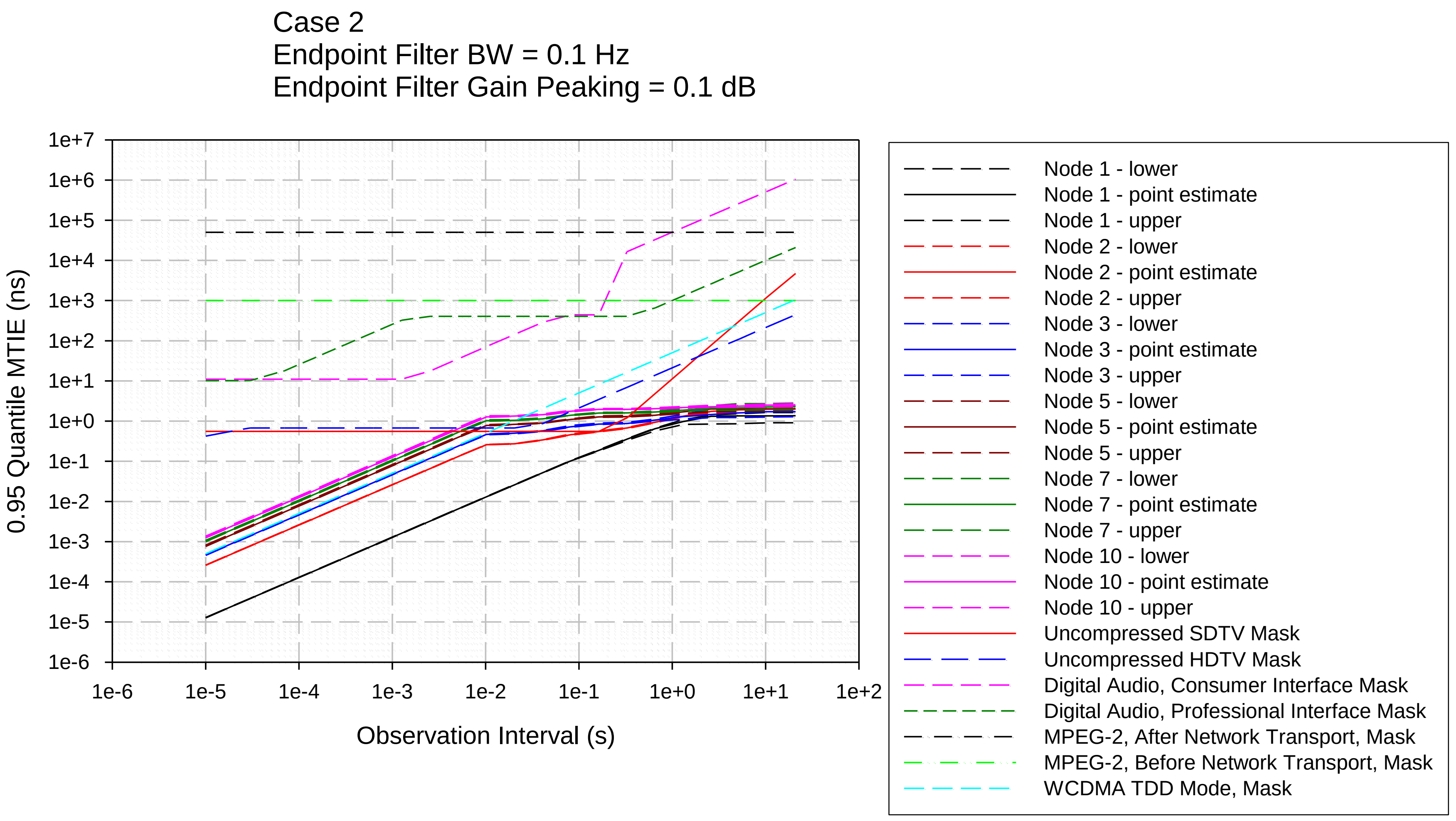
- Jitter and wander requirements can be expressed in terms of Maximum Time Interval Error (MTIE) masks
- MTIE is peak-to-peak phase variation for a specified observation interval, expressed as a function of the observation interval
  - An estimate of MTIE may be computed by (see [6])

$$\mathrm{MTIE}(n\tau_0) \cong \max_{1 \le k \le N-n} \left(\max_{k \le i \le k+n} x(i) - \min_{k \le i \le k+n} x(i) \right), \quad n = 1, 2, \ldots, N-1$$

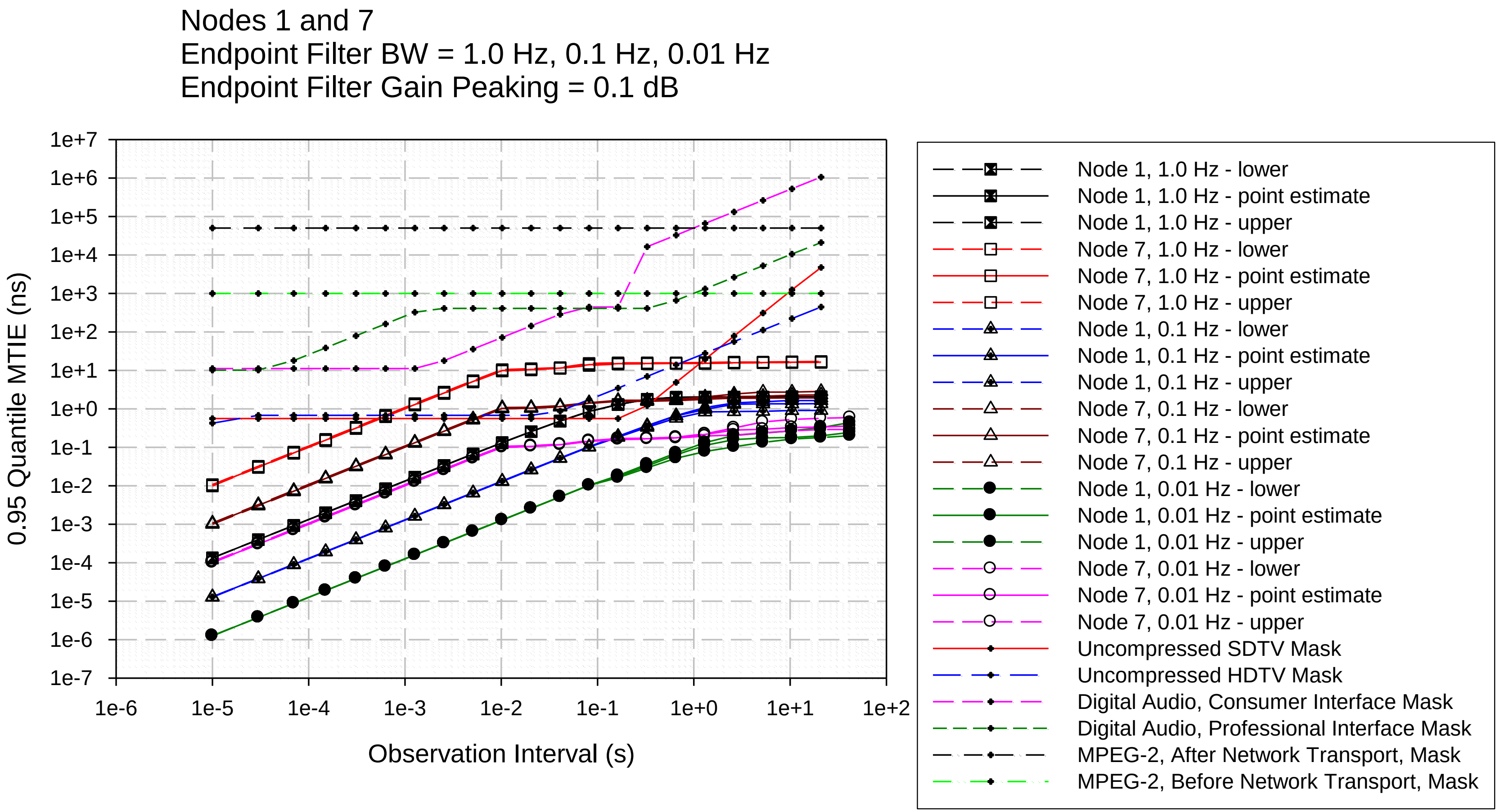
where τ_0 is the sampling interval, $n\tau_0$ is the observation interval, $x(i)$ is the i^{th} phase sample, and N is the number of phase samples ($N\tau_0$ is the measurement interval)

- The derivation of the MTIE mask on slide 5 from the jitter and wander requirements on slide 4 is given in [1]

Appendix III - Case 2 Simulation Results



Appendix III - Comparison of Results - Nodes 1 and 7



Appendix III – Notes on results -1

- Consider a linear, 2nd order filter with undamped natural frequency ω_n, damping ratio ζ, and 20 dB/decade roll-off
  - The transfer function is given by

$$H(s) = \frac{2\varsigma\omega_n s + \omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

- The impulse response may be obtained by taking the inverse Laplace Transform of the transfer function; the result is

$$h(t) = \omega_n e^{-\varsigma\omega_n t}\left[2\varsigma \cosh\sqrt{\varsigma^2 - 1}\,\omega_n t + \frac{1-2\varsigma^2}{\sqrt{\varsigma^2 - 1}}\sinh\sqrt{\varsigma^2 - 1}\,\omega_n t\right]$$

- The impulse response of this filter takes on its maximum value at time zero
  - The maximum value is equal to $2\zeta\omega_n$
- Since the phase measurement is always truncated to the next lower multiple of 40 ns, the filtered phase error contribution of one node is equal, in worst case, to $2\zeta\omega_n(40\text{ ns})$

Appendix III – Notes on results -2

- Can use this as a rule of thumb to conservatively estimate the MTIE contribution from phase measurement granularity at one node
- For example, for a 0.1 Hz filter with 0.1 dB gain peaking, the undamped natural frequency and damping ratio are 0.071781 rad/s and 4.3188, respectively
  - The filtered phase error due to a 40 ns impulse is 0.62 ns
  - Long-term MTIE for 10 nodes for this case is approximately 2.8 ns for the 0.95 quantile estimate based on 300 runs (slide 29)
    - The 300 runs also did not produce any instances where the pulses from all 10 nodes lined up in time
    - If the pulses from all 10 nodes lined up in time, the resulting long-term MTIE for node 10 would be on the order of 6.2 ns
- Note that while the rule of thumb gives an estimate of long-term MTIE, it does not give MTIE as a function of observation interval and, in particular, short observation intervals
  - Simulation is necessary to obtain the actual MTIE performance for various observation intervals, and also to include other effects (e.g., errors in propagation delay measurement, errors due to asymmetries in cable delay and PHY latency)