ITSF keynote address
Clocks for Synchronization

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www.allanstime.com

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The Gold is in the Long-term

Pioneering work of Mandelbrot and Voss

- Developed FRACTILES
- Demonstrated mathematically and pictorially that the world can be well modeled by self-similar and flicker-noise like processes
- Time and frequency community had been working, similarly, on long-term model
- This has very important implications for metrology (non-stationary statistics)

Fundamental Time-Domain Problem

- Classical variance or Standard deviation does not converge for several important noise processes in clocks, navigation, and telecommunication systems.
- Classical variance cannot distinguish the different important kinds of noise relevant to clocks, oscillators, and to navigation and telecommunication systems.

Standard Deviation
is like
black and white

AVAR, MVAR and TVAR
give intensity and color

Characteristics of Useful Measures

- Theoretically Sound
- Easy to use and intuitive
- Relates to real situations
- Yields useful spectral information for design engineers
- Useful diagnostic tool
- Optimum smoothing, estimation, and prediction
- Communicates to the Manager (Decision Makers)

HISTORICAL PERSPECTIVE

- 1964 IEEE & NASA Short-term Stability Symposium
- 1966 IEEE Proceedings, Feb. Special Issue: "Frequency Stability"
- 1971 IEEE I&M, *Characterization of Frequency Stability*
- 1974 NBS Monograph 140
- 1981 Frequency Control Symposium: *Modified "Allan Variance" with Increased Oscillator Characterization Ability*
- 1988 IEEE Standard 1139-1988: *Standard Terminology for Fundamental Frequency and Time Metrology*
- Late 1980s: development of TVAR (Time Variance for telecom.)
- 1990 NIST Technical Note 1337, *Characterization of Clocks and Oscillators*
- 1997 ITU HANDBOOK: "Selection and Use of Precise Frequency and Time Systems," Radiocommunication Bureau
- 1997 Hewlett Packard Application Note 1289, *The Science of Timekeeping*
- 2000-2010 Additional variance work at NIST provides efficiency & tighter confidences – http://tf.nist.gov/general/publications.htm
- Handbook of Frequency Stability Analysis, by W. J. Riley, NIST Special Publication, SP 1065 (2007); also available at www.wriley.com

Time - Domain Measures

- ***Frequency Accuracy:***
  The degree of conformity with a standard or a definition.

- ***Frequency Instability:***
  Change, typically averaged over an interval, τ, with respect to another frequency.

- ***Time Accuracy:***
  The degree of conformity with UTC or some agreed upon time-scale.

- ***Time Instability:***
  Change, in residual readings, typically averaged over an interval, τ, with respect to nominal or other averaged interval(s).

$y(t)$ $x(t)$ $\sigma_y(\tau)$ $\sigma_x(\tau)$ $Mod.\,\sigma_y(\tau)$ UTC

Definitions

- Synchronization – Two clocks are synchronized if they read the same time (accounting for propagation delays and relativistic effects)
- Syntonization – Two clocks are syntonized if they are running at the same rate in a particular frame of reference (their times can be very different)

Define: Normalized Freq. & Time Residual

$$y(t) = \frac{\nu(t) - \nu_o}{\nu_o}$$

Dimensionless Normalized Freq.

$$x(t) = \frac{\varphi(t)}{2\pi \nu_o}$$

Time Residual

Then $$y(t) = \frac{dx(t)}{dt} \equiv \dot{x}(t)$$

And $$x(t) = \int_o^t y(t')\, dt'$$

5-6

FLIP OF A COIN IS A RANDOM UNCORRELATED PROCESS:
(white noise spectrum)

$$S_y(f) \propto f^{0}$$

INTEGRATING THESE FLIPS GENERATES A RANDOM-WALK PROCESS:
(heads = one step forward)
(tails = one step backward)
AFTER “N” FLIPS OF A COIN, WILL BE (on average) √N AWAY FROM THE ORIGIN:

$$S_x(f) \propto f^{-2}$$

Since: y = dx/dt, or

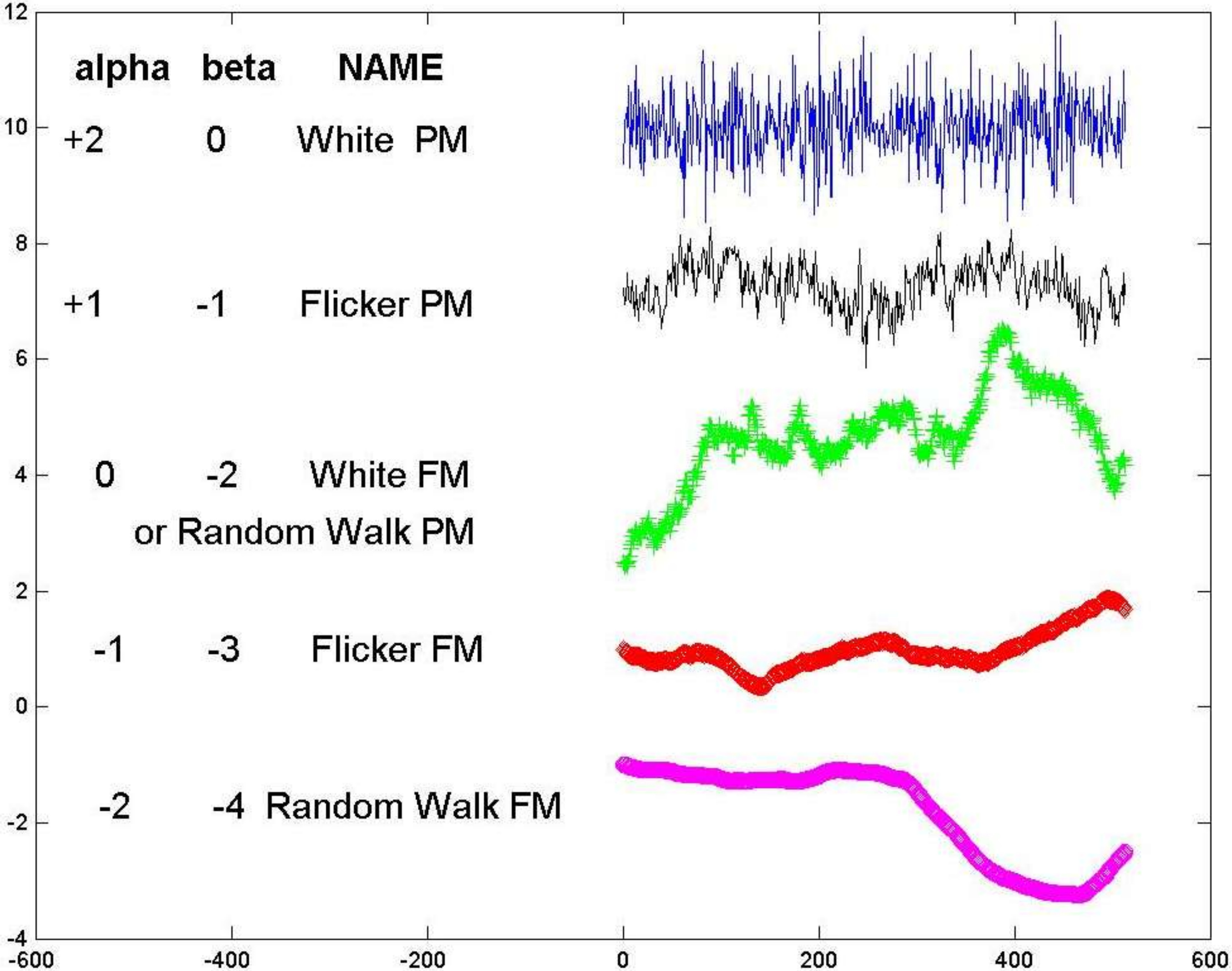
$$x(t) = \int_0^t y(t')dt'$$

Similarly, taking a first difference is like a derivative and turns a random-walk process into a white-noise process.

$$y_i = \frac{\Delta x}{\tau} = \frac{x_{i+1} - x_i}{\tau}$$

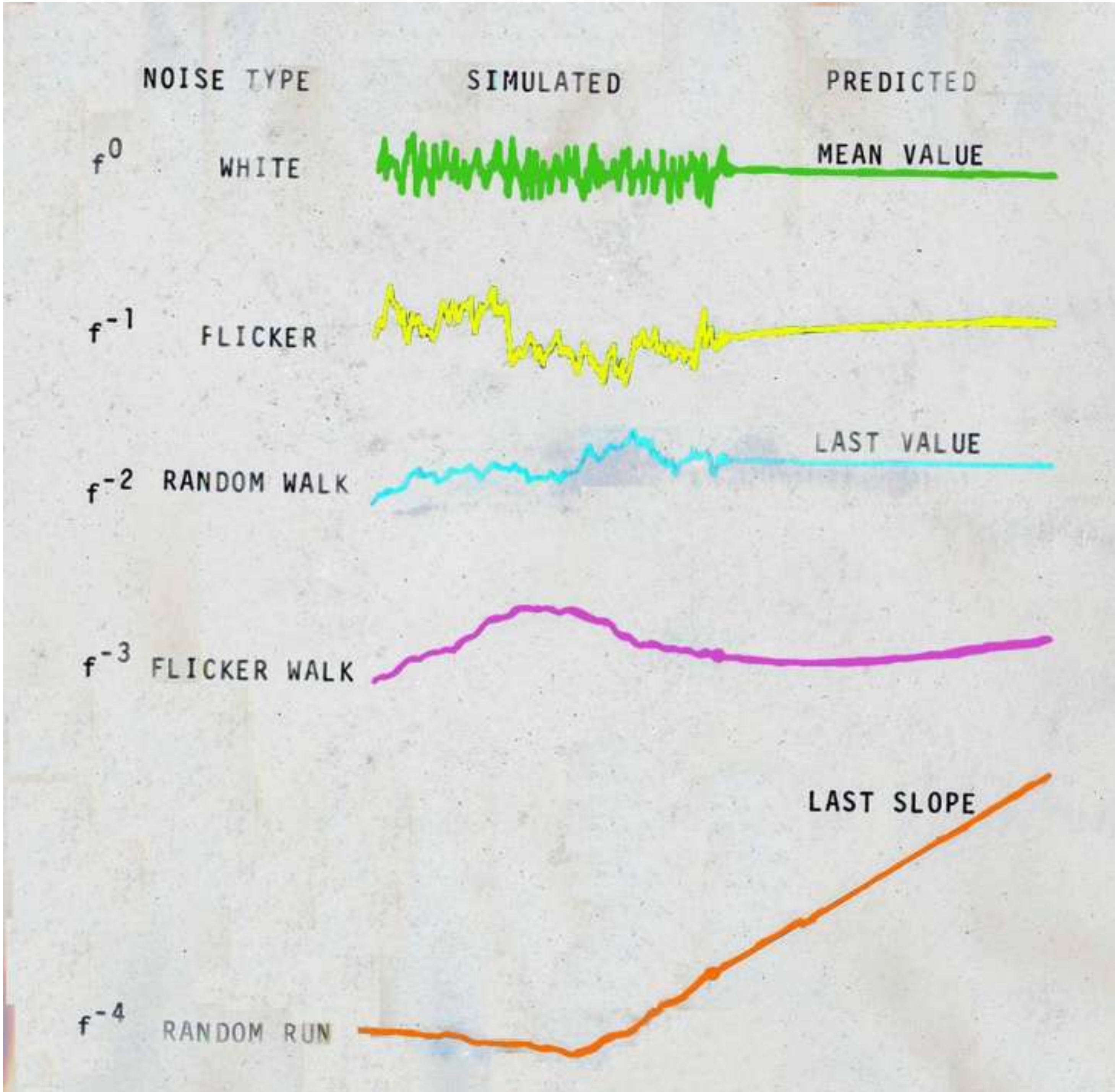
POWER-LAW SPECTRA:

$$S_y(f) \propto f^{\alpha} \qquad S_x(f) \propto f^{\beta}$$

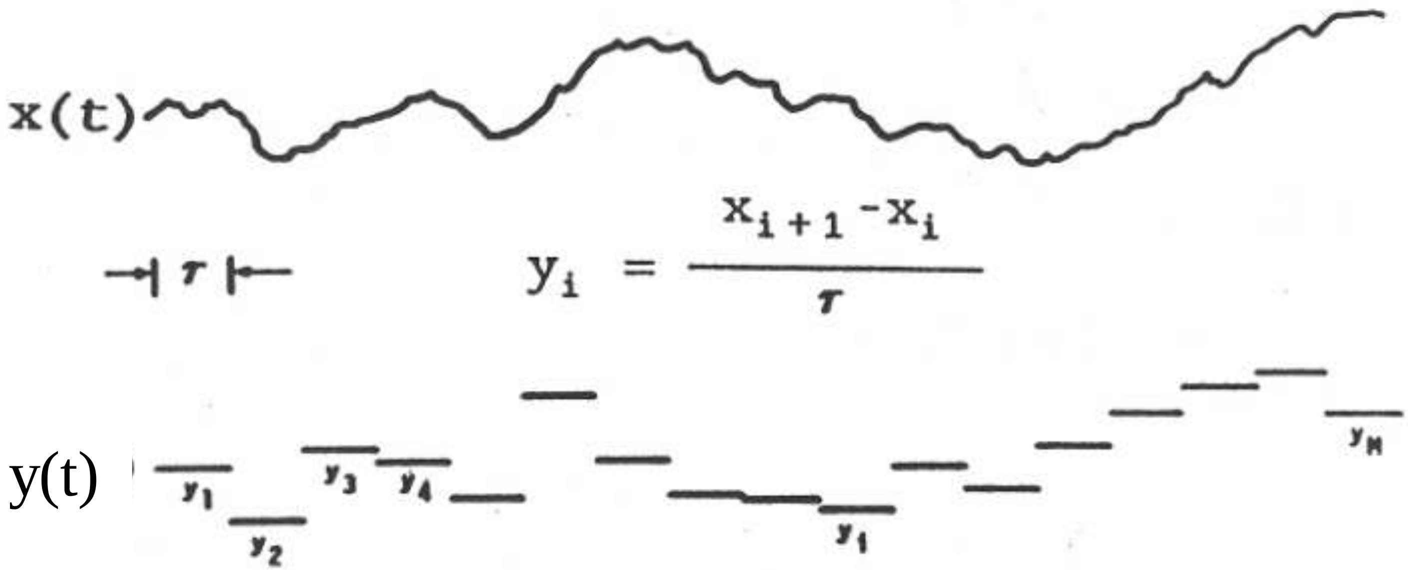


STATISTICAL THEOREM

- The optimum estimate of the mean of a process with a white-noise spectrum is the simple mean.
- HENCE:
  - For white PM, the optimum estimate of the phase or the time is the simple mean of the independent phase or time readings.
  - For White FM, the optimum estimate of the frequency is the simple mean of the independent frequency readings, which is equivalent to the last time reading minus the first time reading divided by the data length, if there is no dead-time between the readings.


NOISE TYPE
SIMULATED
PREDICTED
f^0 WHITE
MEAN VALUE
f^{-1} FLICKER
LAST VALUE
f^{-2} RANDOM WALK
f^{-3} FLICKER WALK
LAST SLOPE
f^{-4} RANDOM RUN

GIVEN THE TIME RESIDUALS FROM A PRECISSION CLOCK OR OSCILLATOR.



$$\sigma_{STD\ DEV\ y}(\tau) = \sqrt{\frac{1}{M-1}\sum_{i=1}^{M}(y_i - \bar{y})^2}$$

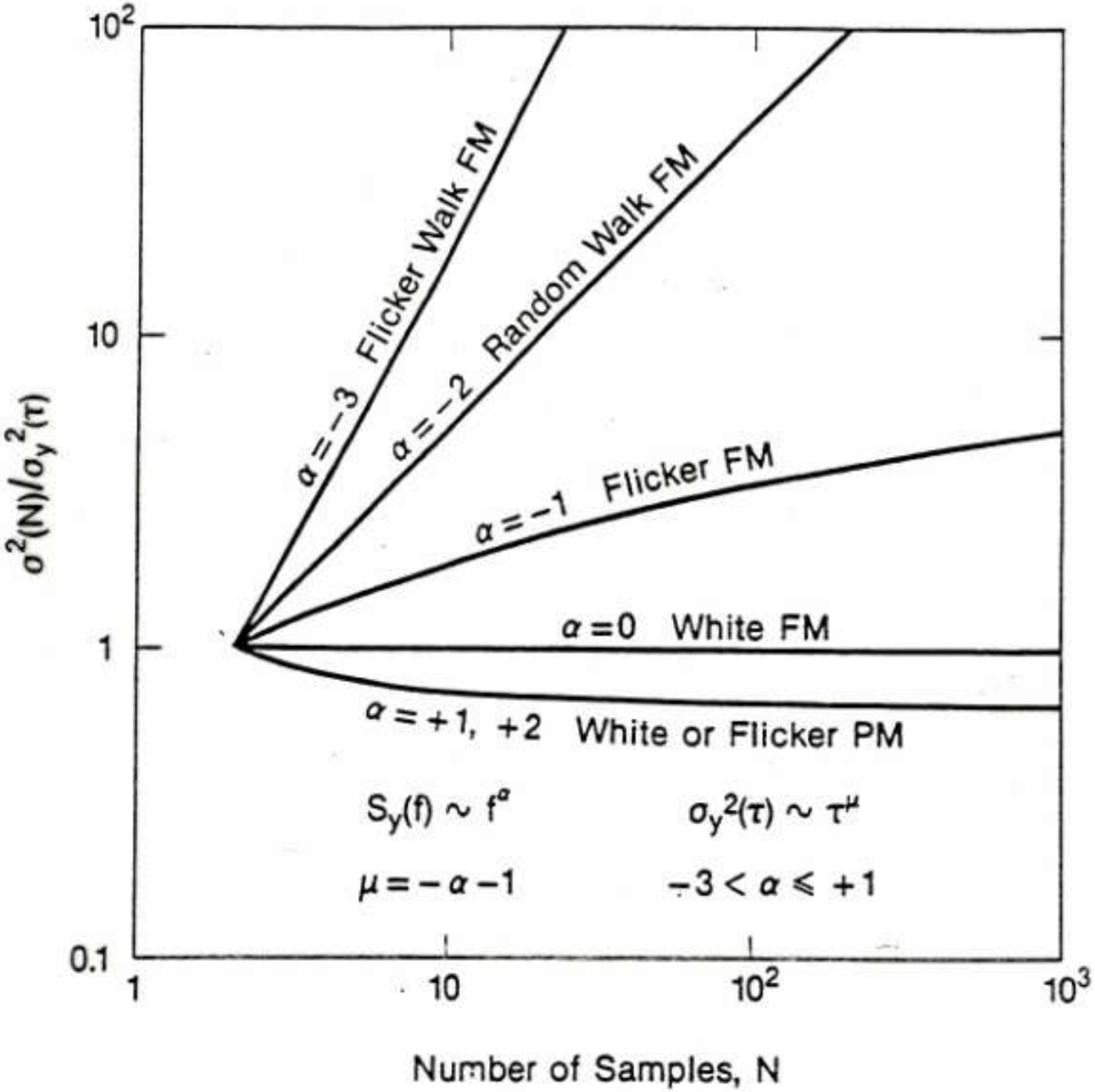
↑

DOES NOT CONVERGE AS M INCREASES

$$\sigma_y(\tau) = \sqrt{\frac{1}{2(M-1)}\sum_{i=1}^{M-1}(y_{i+1} - y_i)^2}$$

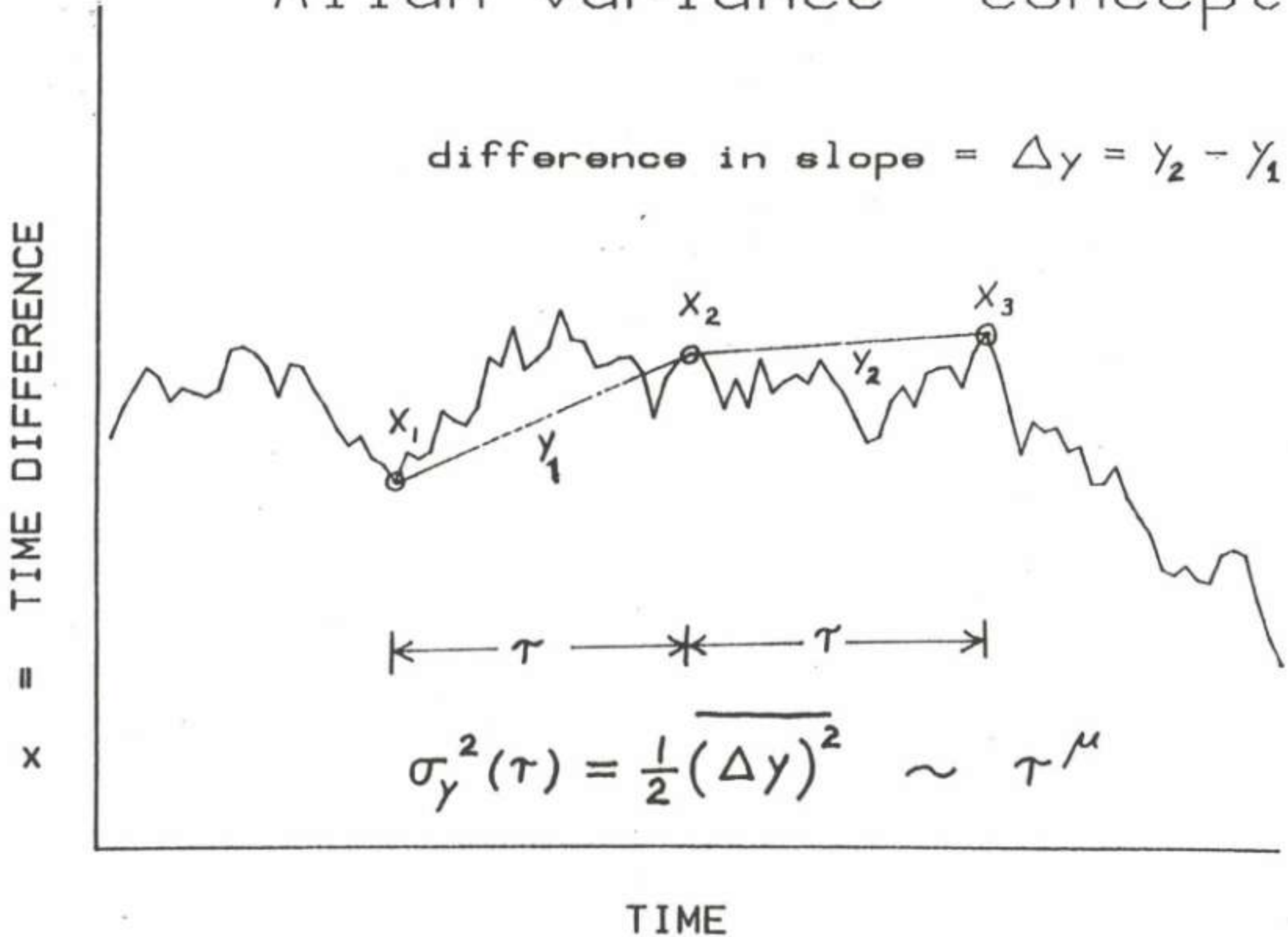
↑

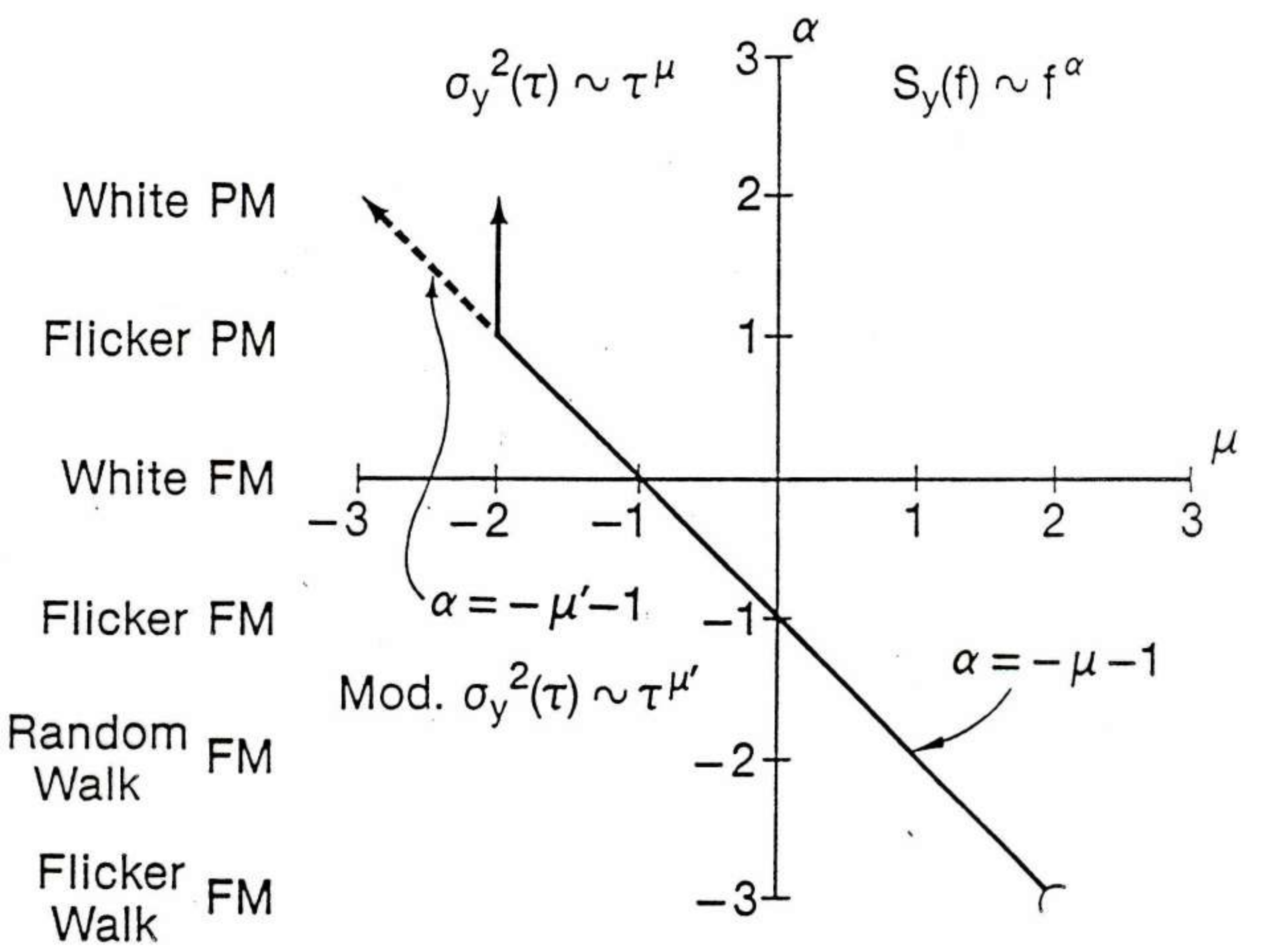
DOES CONVERGE AS M INCREASES



10^2
10
1
0.1
$\sigma^2(N)/\sigma_y^2(\tau)$
$\alpha = -3$ Flicker Walk FM
$\alpha = -2$ Random Walk FM
$\alpha = -1$ Flicker FM
$\alpha = 0$ White FM
$\alpha = +1, +2$ White or Flicker PM
$S_y(f) \sim f^\alpha$
$\sigma_y^2(\tau) \sim \tau^\mu$
$\mu = -\alpha - 1$
$-3 < \alpha \leqslant +1$
1
10
10^2
10^3
Number of Samples, N

'Allan variance' concept





$\sigma_y^2(\tau) \sim \tau^{\mu}$
$S_y(f) \sim f^{\alpha}$
White PM
Flicker PM
White FM
Flicker FM
Random Walk FM
Flicker Walk FM
α
μ
3
2
1
−1
−2
−3
−3
−2
−1
1
2
3
$\alpha = -\mu' - 1$
Mod. $\sigma_y^2(\tau) \sim \tau^{\mu'}$
$\alpha = -\mu - 1$

SINCE

$$\sigma_y^2(\tau) \sim \tau^{\mu},$$

THEN

$$\sigma_y(\tau) \sim \tau^{\mu/2}.$$

THE LOG OF THIS IS

$$\mathrm{LOG}(\sigma_y(\tau)) \sim \mu/2 \ \mathrm{LOG}(\tau);$$

THEREFORE, THE SLOPE ON A LOG LOG PLOT OF $\sigma_y(\tau)$ VS. τ IS $\mu/2$

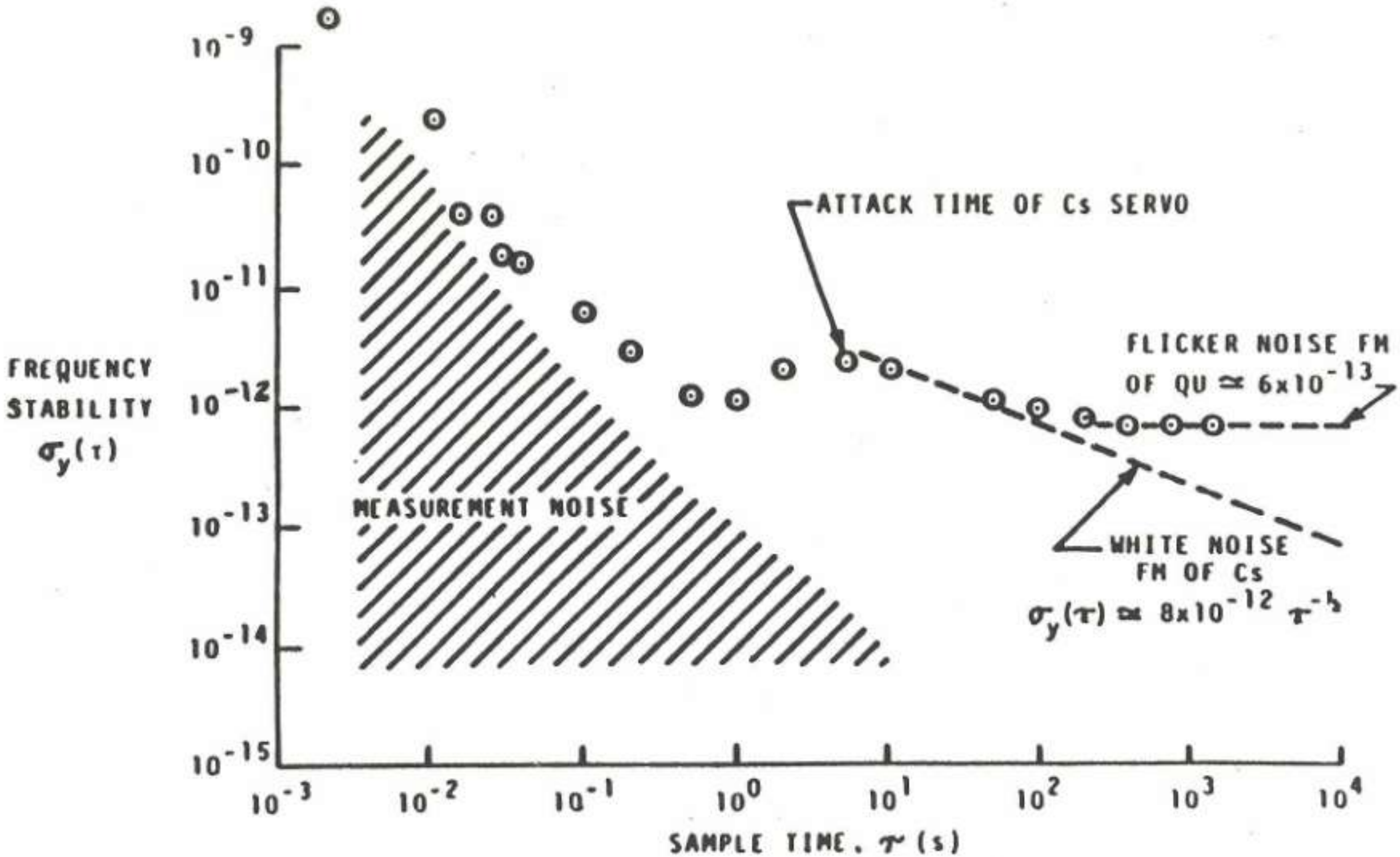
SINCE

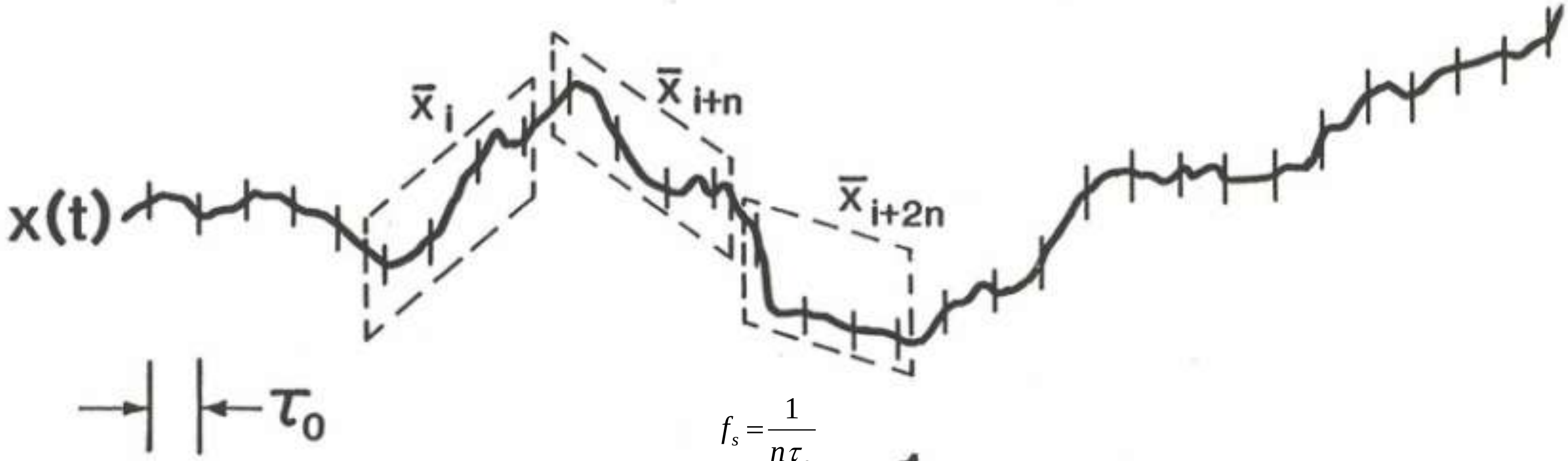
$$\alpha = -\mu - 1,$$

$$(-2 \leq \mu < 2)$$

WE CAN DEDUCE THE SPECTRAL TYPE OF NOISE KNOWING α

QUARTZ OSCILLATOR (DIANA) VS COMMERCIAL CESIUM (#601)





$$f_s = \frac{1}{n\tau_o}$$

$$\text{Mod. } \sigma_y^2(\tau) = \frac{1}{2\tau^2}\langle(\Delta^2 \bar{x}_i)^2\rangle$$

$$\tau = n\tau_0$$

$$f_s = \frac{1}{n\tau_o}$$

Frequency Domain Spectral Densities

$S_y(f)$, $S_\phi(f)$, $S_{\dot\phi}(f)$, or $S_x(f)$

Relationships:

$$S_y(f) = \frac{f^2}{\nu_0^2} S_\phi(f)$$

$$S_{\dot\phi}(f) = (2\pi f)^2 S_\phi(f)$$

$$S_x(f) = \frac{1}{(2\pi\nu_0)^2} S_\phi(f)$$

$$S_y(f) = (2\pi f)^2 S_x(f).$$

Time Domain Variances

AVAR:

$$\sigma_y^2(\tau) = \frac{1}{2} \langle(\bar{y}_{k+1} - \bar{y}_k)^2\rangle$$

$$= \frac{1}{2\tau^2} \langle(x_{k+2} - 2x_{k+1} + x_k)^2\rangle$$

MVAR:

$$\text{mod.}\sigma_y^2(\tau) = \frac{1}{2\tau^2} \langle(\bar{x}_{k+2} - 2\bar{x}_{k+1} + x_k)^2\rangle$$

TVAR:

$$\sigma_x^2(\tau) = \frac{\tau^2}{3} \text{mod.}\sigma_y^2(\tau)$$

$$\sigma_y^2(\tau) = \frac{1}{2}\left\langle (\Delta \bar{y})^2 \right\rangle$$

$$\bar{y} = \frac{x(t+\tau) - x(t)}{\tau}$$

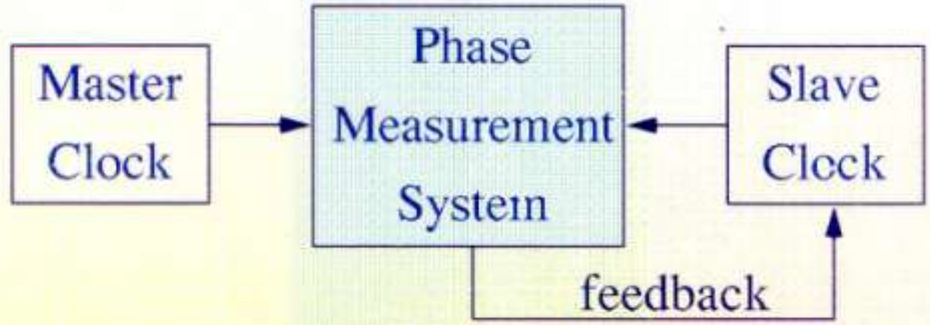
is optimum for white FM

$$\sigma_x^2(\tau) = \frac{1}{6}\left\langle (\Delta^2 \bar{x})^2 \right\rangle$$

$\bar{x}$ = avg. of x over τ

is optimum for white PM

Generic Phase-lock Loop



Fundamental Properties

$$y = \frac{\nu - \nu_0}{\nu_0} \qquad x = \int y'\, dt' = \frac{\phi}{2\pi\nu_0} \qquad y$$

Measures

$\sigma_y(\tau)$	$\sigma_x(\tau)$	$\sigma_y(\tau)$
mod $\sigma_y(\tau)$	$S_x(f)$	mod $\sigma_y(\tau)$
$S_y(f)$	$S_\phi(f)$	$S_y(f)$
$S_\phi(f)$		$S_\phi(f)$

Clock-time Keeping Ability

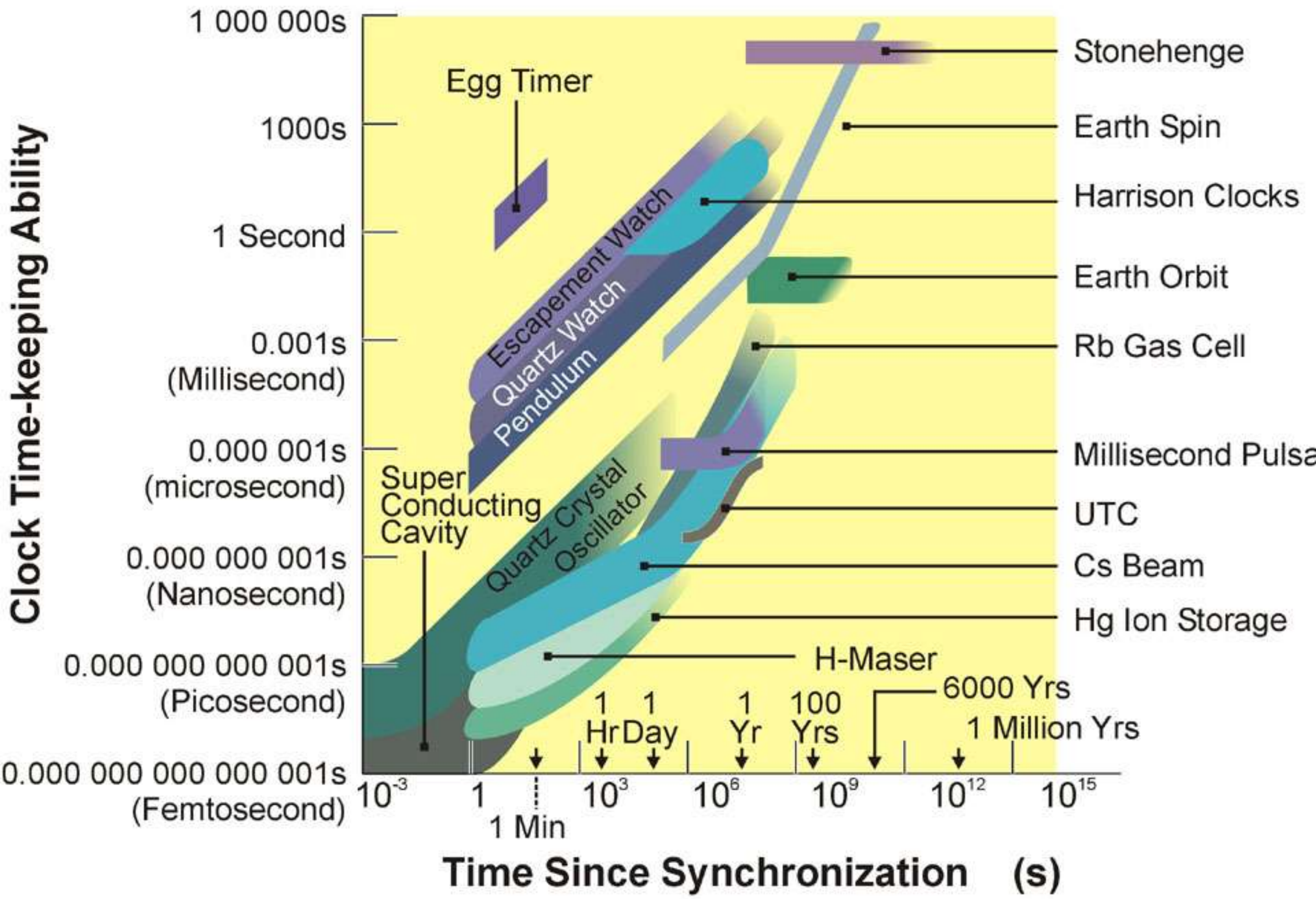
$$x_p(\tau) = k\ \tau\ \sigma_y(\tau)$$

$$k = 1 \ \ for\ white\ FM\ and\ random\ walk\ FM$$

$$k = 1/\sqrt{\ln 2} = 1.2 \quad for\ \ flic\,\mathrm{ker}\,FM$$

$$k = 1/\sqrt{3} \quad for\ \ white\ PM\ and\ \ flic\,\mathrm{ker}\,PM$$

Time Dispersion of Various Clocks



Translation Between Frequency and Time Domains

$$\tau = n\tau_0$$

AVAR:

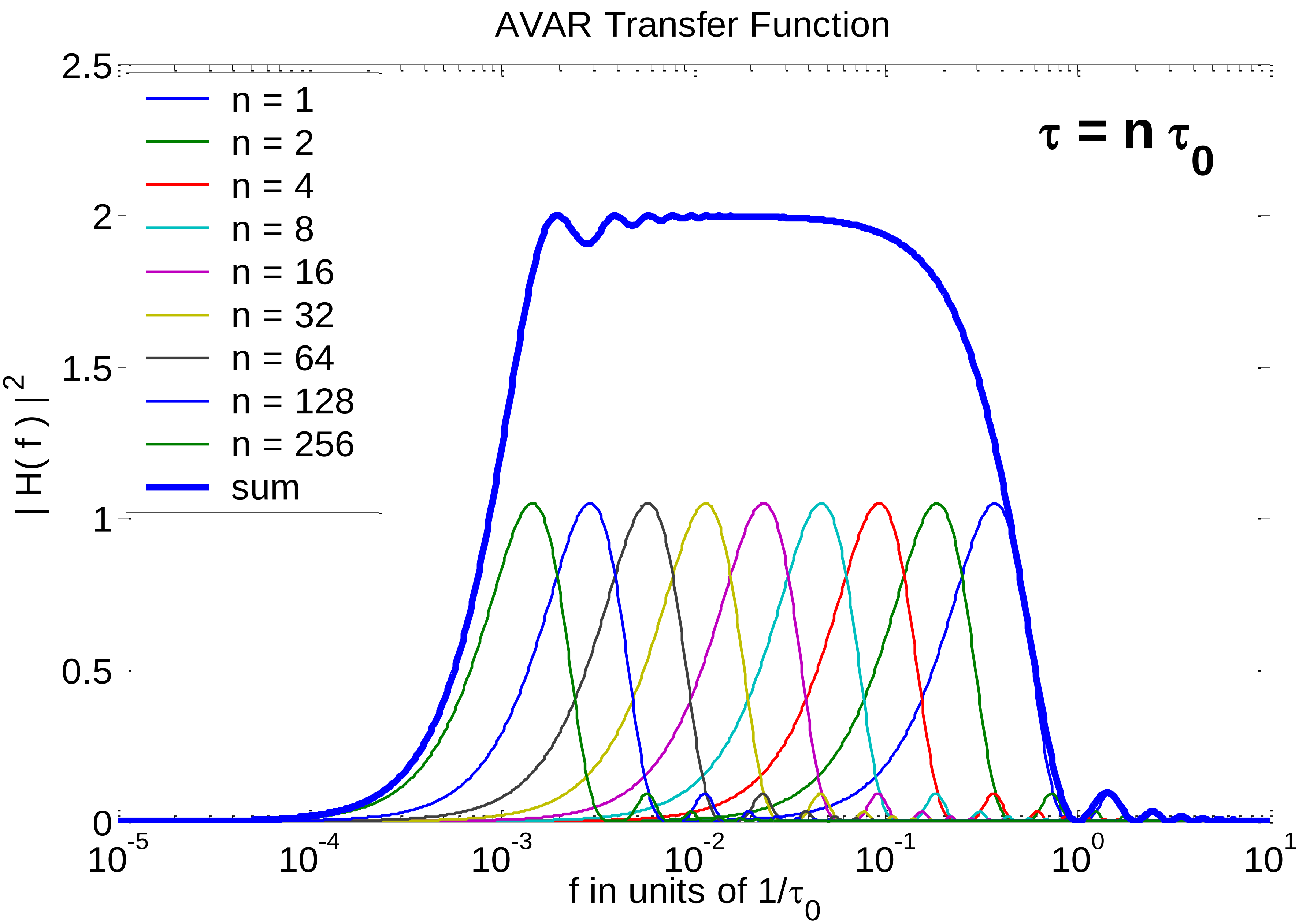
$$\sigma_y^2(\tau) = \int_0^\infty 2\left[\frac{\sin^4(\pi f\tau)}{(\pi f\tau)^2}\right] S_y(f)df$$

MVAR:

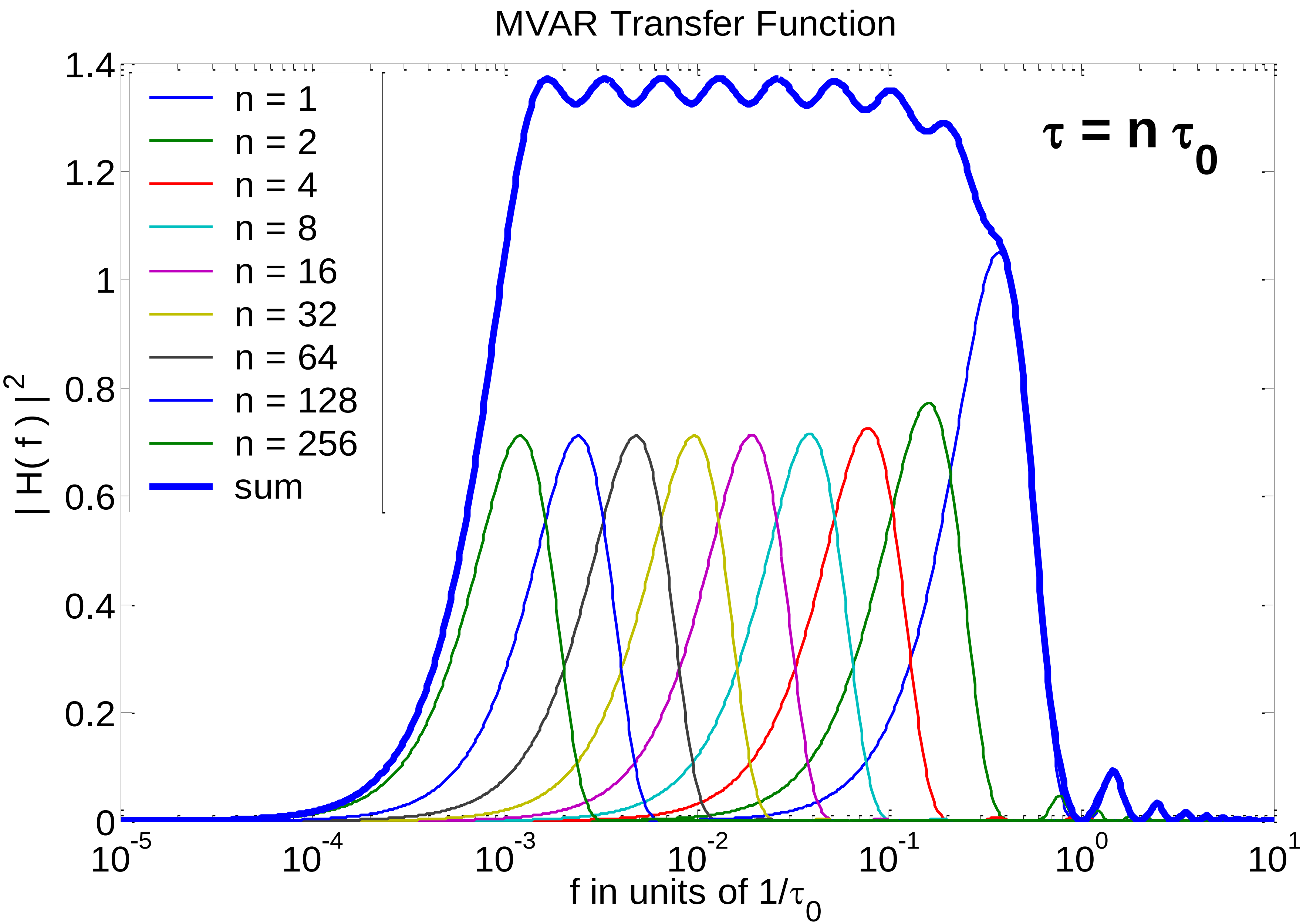
$$\mathrm{Mod}\,\sigma_y^2(\tau) = \int_0^\infty 2\left[\frac{\sin^3(\pi f\tau)}{(n\pi f\tau)\sin(\pi f\tau_0)}\right]^2 S_y(f)df$$

TVAR:

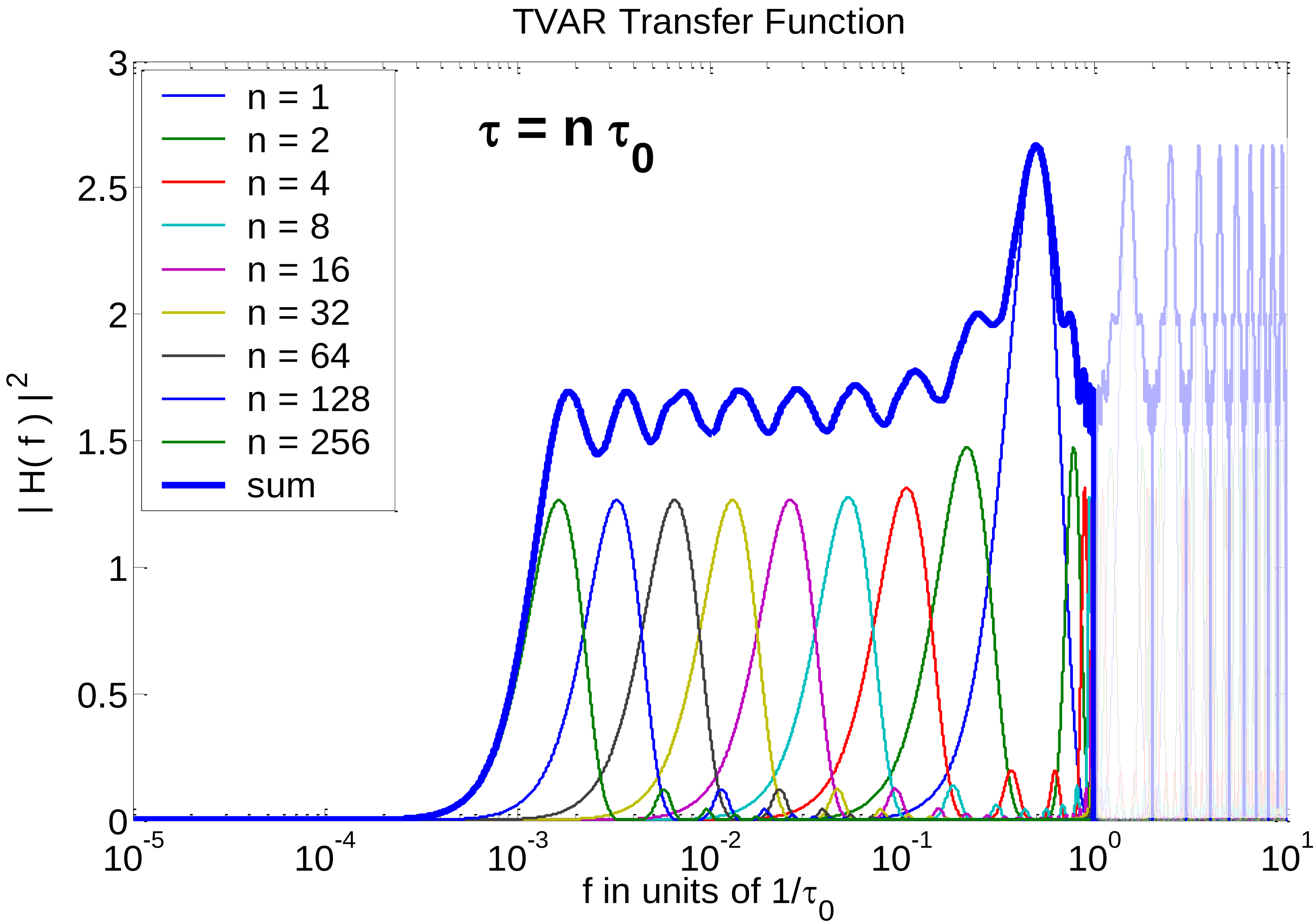
$$\sigma_x^2(\tau) = \frac{8}{3n^2}\int_0^\infty \left[\frac{\sin^3(\pi f\tau)}{\sin(\pi f\tau_0)}\right]^2 S_x(f)df$$



AVAR Transfer Function
τ = n τ0
n = 1
n = 2
n = 4
n = 8
n = 16
n = 32
n = 64
n = 128
n = 256
sum
| H(f) |2
f in units of 1/τ0
2.5
2
1.5
1
0.5
0
10-5
10-4
10-3
10-2
10-1
100
101

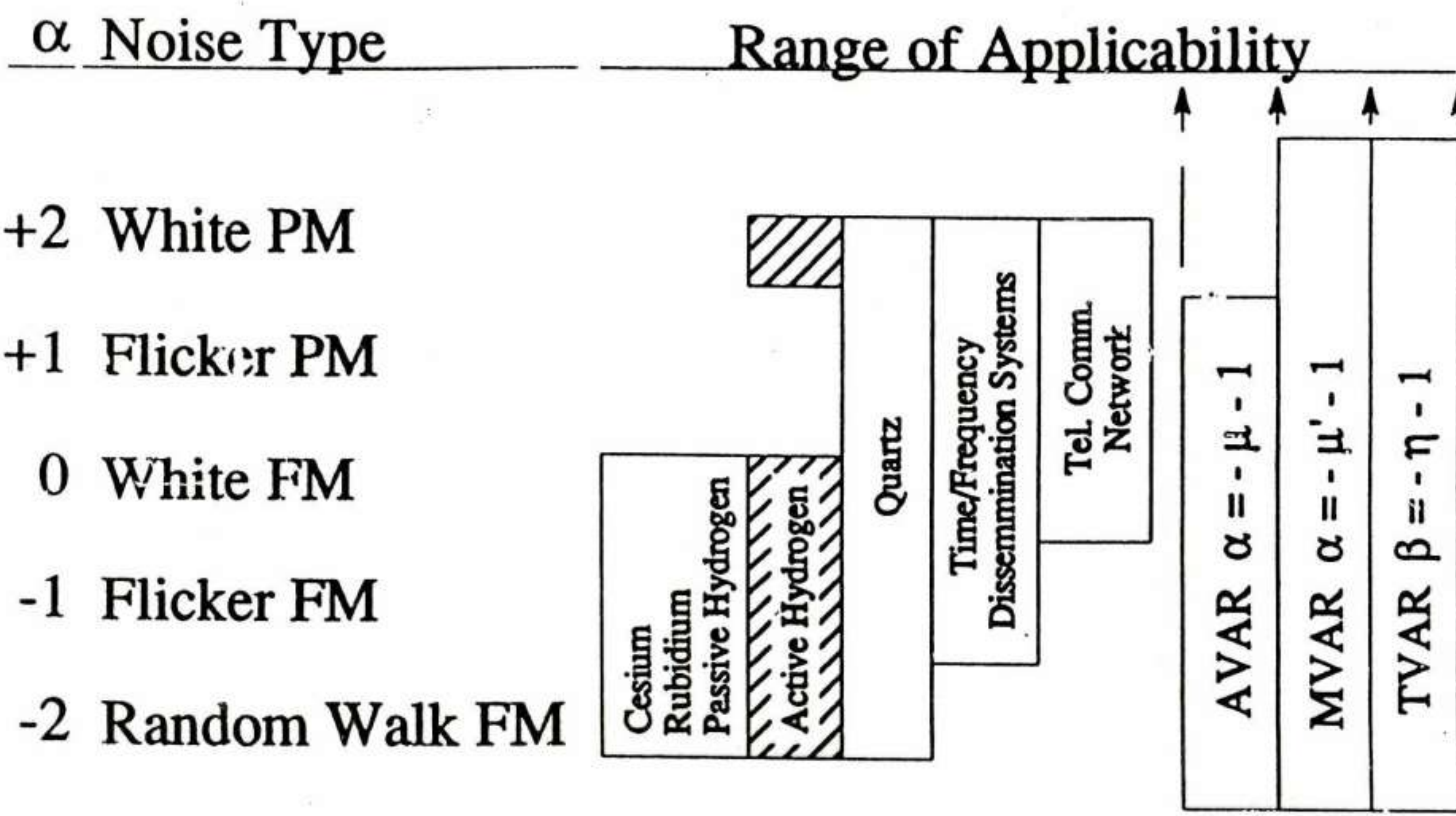


MVAR Transfer Function
1.4
1.2
1
0.8
0.6
0.4
0.2
0
| H(f) |^2
n = 1
n = 2
n = 4
n = 8
n = 16
n = 32
n = 64
n = 128
n = 256
sum
τ = n τ$_0$
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
f in units of 1/τ$_0$


TVAR Transfer Function
τ = n τ0
n = 1
n = 2
n = 4
n = 8
n = 16
n = 32
n = 64
n = 128
n = 256
sum
| H(f) |2
f in units of 1/τ0
0
0.5
1
1.5
2
2.5
3
10-5
10-4
10-3
10-2
10-1
100
101

NOISE TYPE	$S_y(f)$	$S_x(f)$
White PM	$\frac{(2\pi)^2}{3f_h}\,[\tau^2\sigma_y^2(\tau)]\,f^2$	$\frac{1}{\tau_0 f_h}\,[\tau\,\sigma_x^2(\tau)]\,f^0$
Flicker PM	$\frac{(2\pi)^2}{A}\,[\tau^2\sigma_y^2(\tau)]\,f^1$	$\frac{3}{3.37}\,[\tau^0\sigma_x^2(\tau)]\,f^{-1}$
White FM	$2\,[\tau^1\sigma_y^2(\tau)]\,f^0$	$\frac{12}{(2\pi)^2}\,[\tau^{-1}\sigma_x^2(\tau)]\,f^{-2}$
Flicker FM	$\frac{1}{2\ell n2}\,[\tau^0\sigma_y^2(\tau)]\,f^{-1}$	$\frac{20}{(2\pi)^2 9\ell n2}\,[\tau^{-2}\sigma_x^2(\tau)]\,f^{-3}$
Random Walk FM	$\frac{6}{(2\pi)^2}\,[\tau^{-1}\sigma_y^2(\tau)]\,f^{-2}$	$\frac{240}{(2\pi)^4 11}\,[\tau^{-3}\sigma_x^2(\tau)]\,f^{-4}$

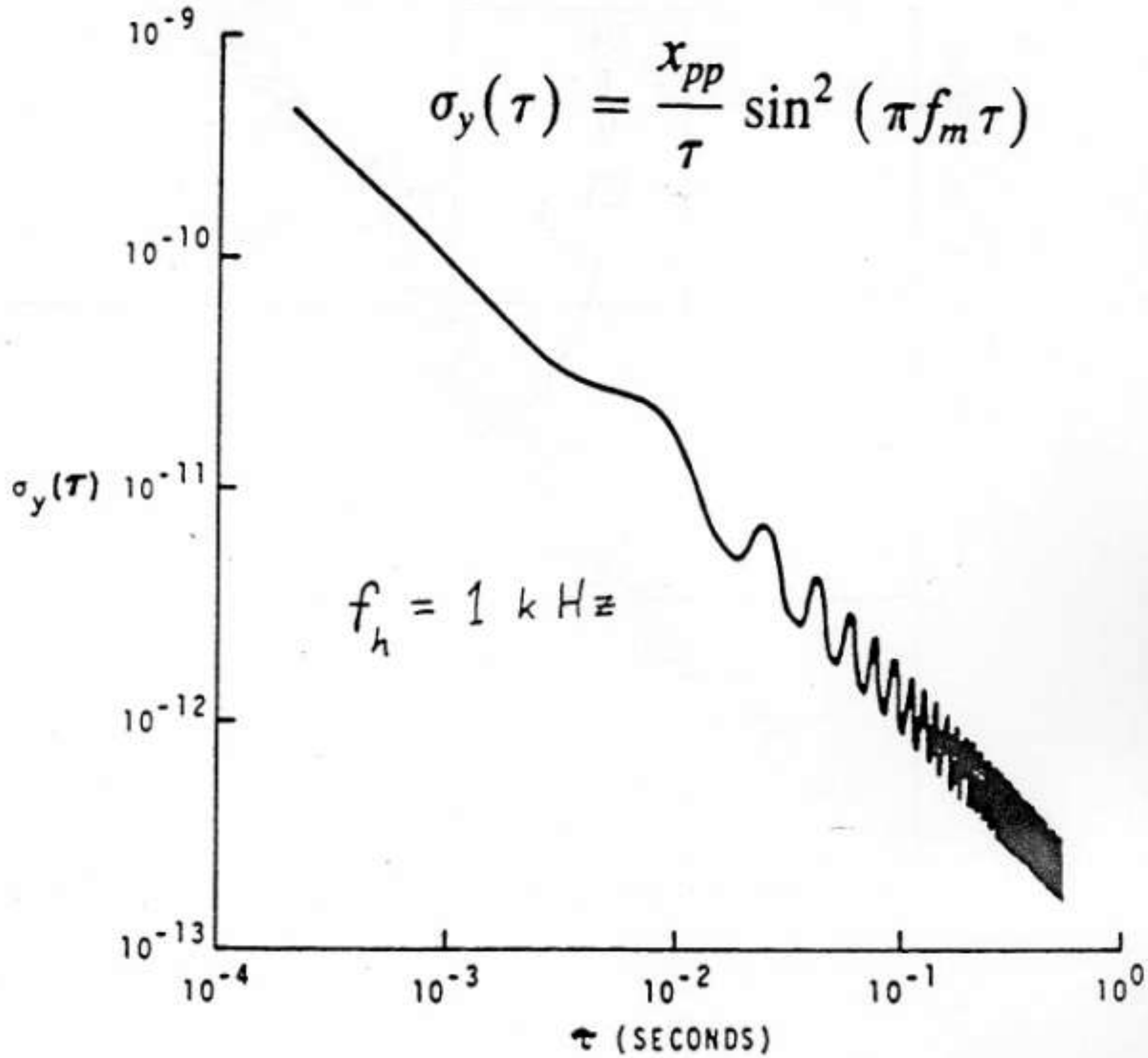
$$A = 1.038 + 3\ell n(2\pi f_h \tau)$$



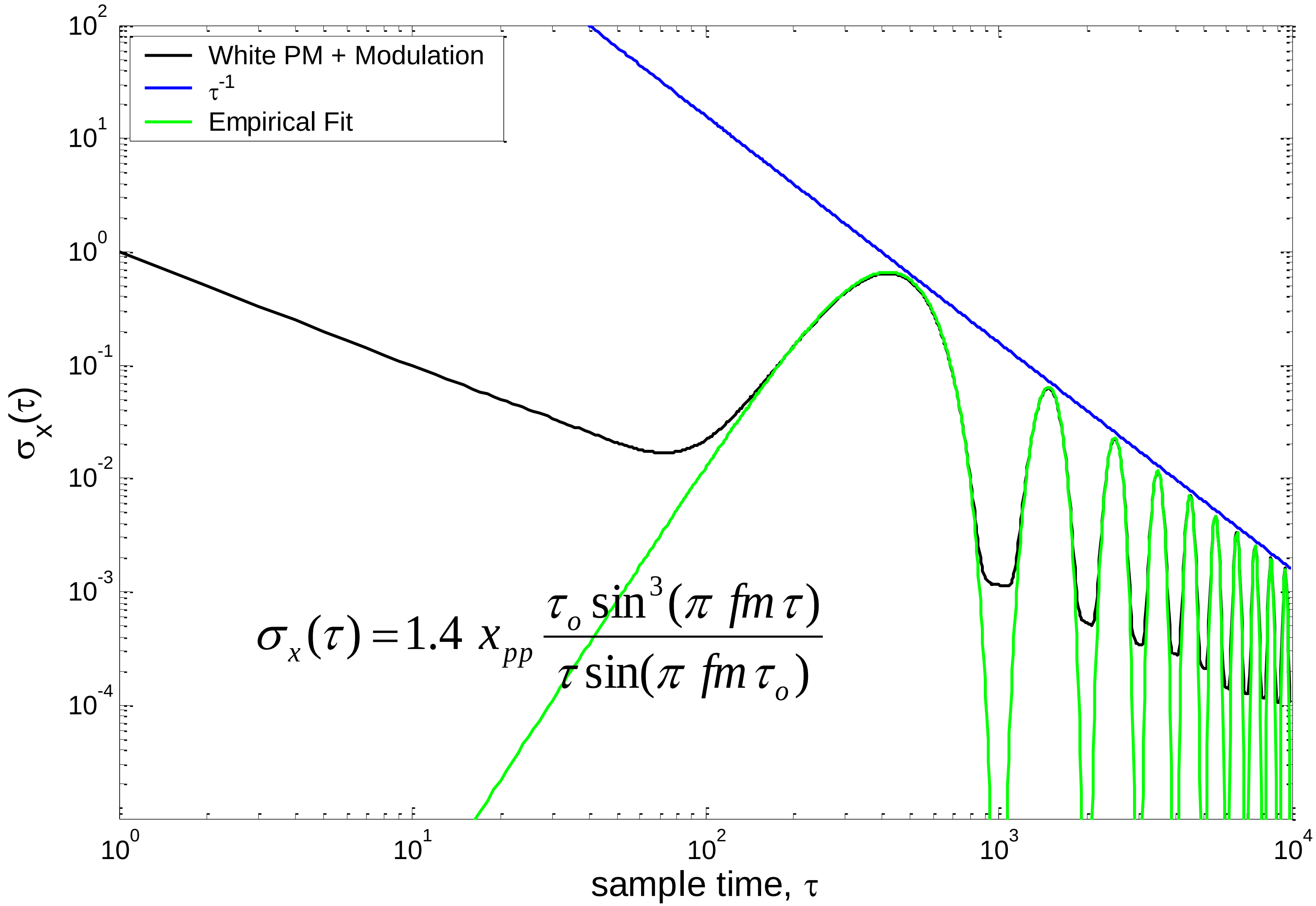
α Noise Type
Range of Applicability
+2 White PM
+1 Flicker PM
0 White FM
-1 Flicker FM
-2 Random Walk FM
Cesium
Rubidium
Passive Hydrogen
Active Hydrogen
Quartz
Time/Frequency
Dissemination Systems
Tel. Comm.
Network
AVAR α = - μ - 1
MVAR α = - μ' - 1
TVAR β = - η - 1

EFFECTS OF SINGLE FREQUENCY MODULATION ON $\sigma_y(\tau)$

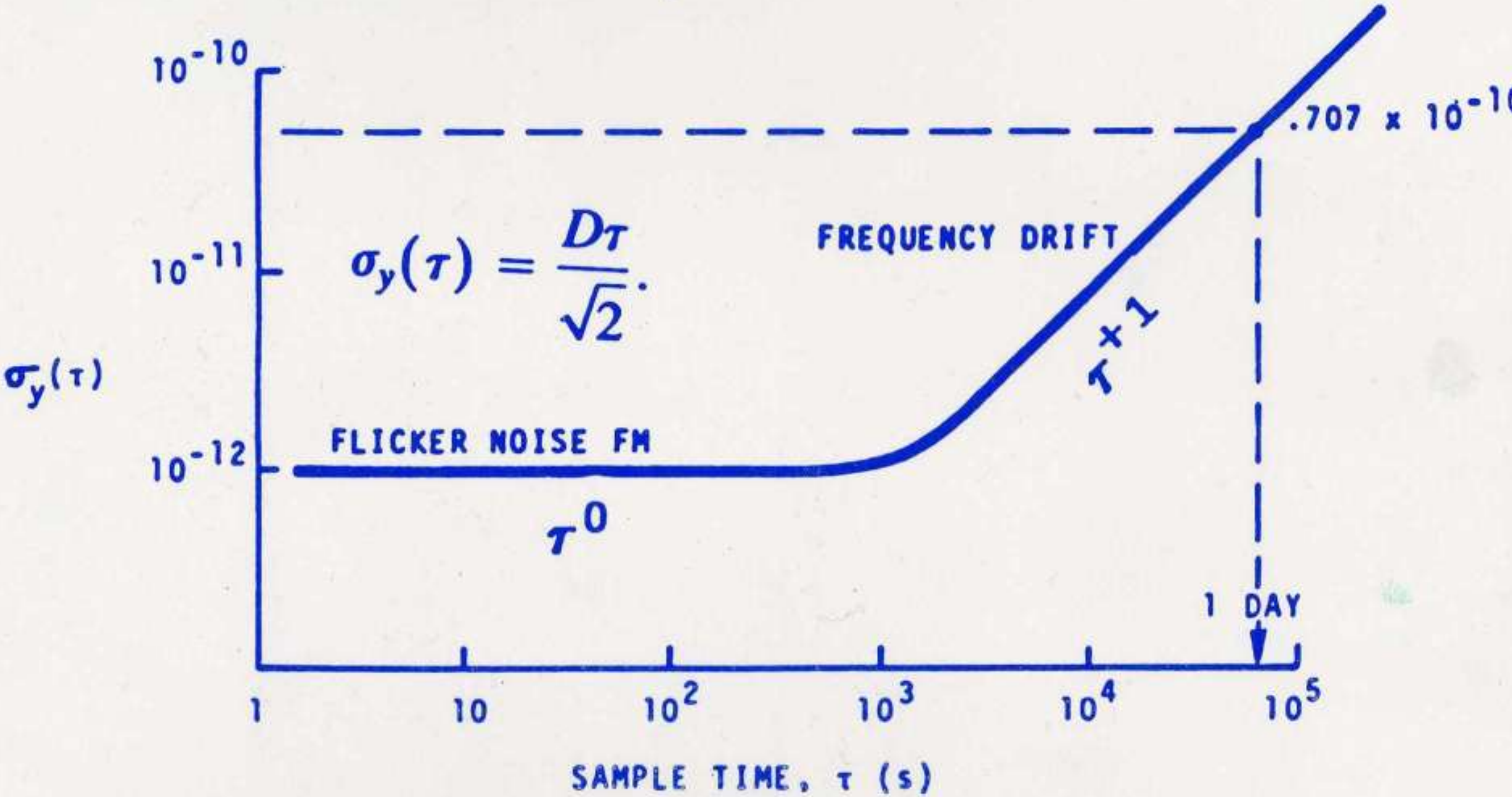
$S_y(f) = 2.5 \times 10^{-15}\,[1+10^3\,\delta(f-60\text{ Hz})]$



Effect of fm on TDEV



EFFECT OF FREQUENCY DRIFT ON $\sigma_y(\tau)$



Time Interval Error's relationship to frequency drift through $\sigma_x(\tau)$.

Given: $x(t) = \frac{1}{2} D t^2$

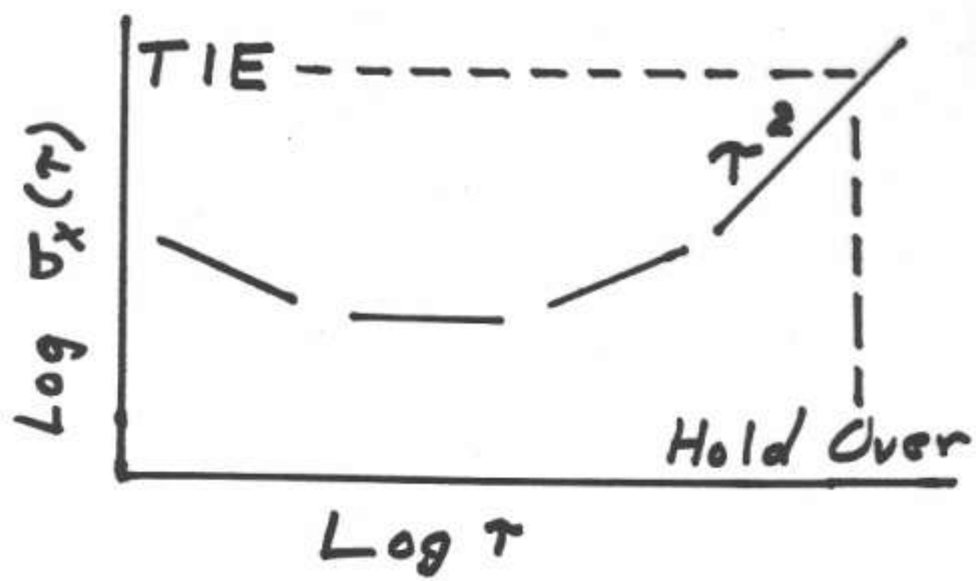
and $\sigma_y(\tau) = \frac{D\tau}{\sqrt{2}}$

It may be shown that

$$\text{Mod. } \sigma_y(\tau) \simeq \frac{D\tau}{\sqrt{2}}$$

Also $\sigma_x(\tau) = \frac{\tau}{\sqrt{3}} \text{ Mod. } \sigma_y(\tau)$

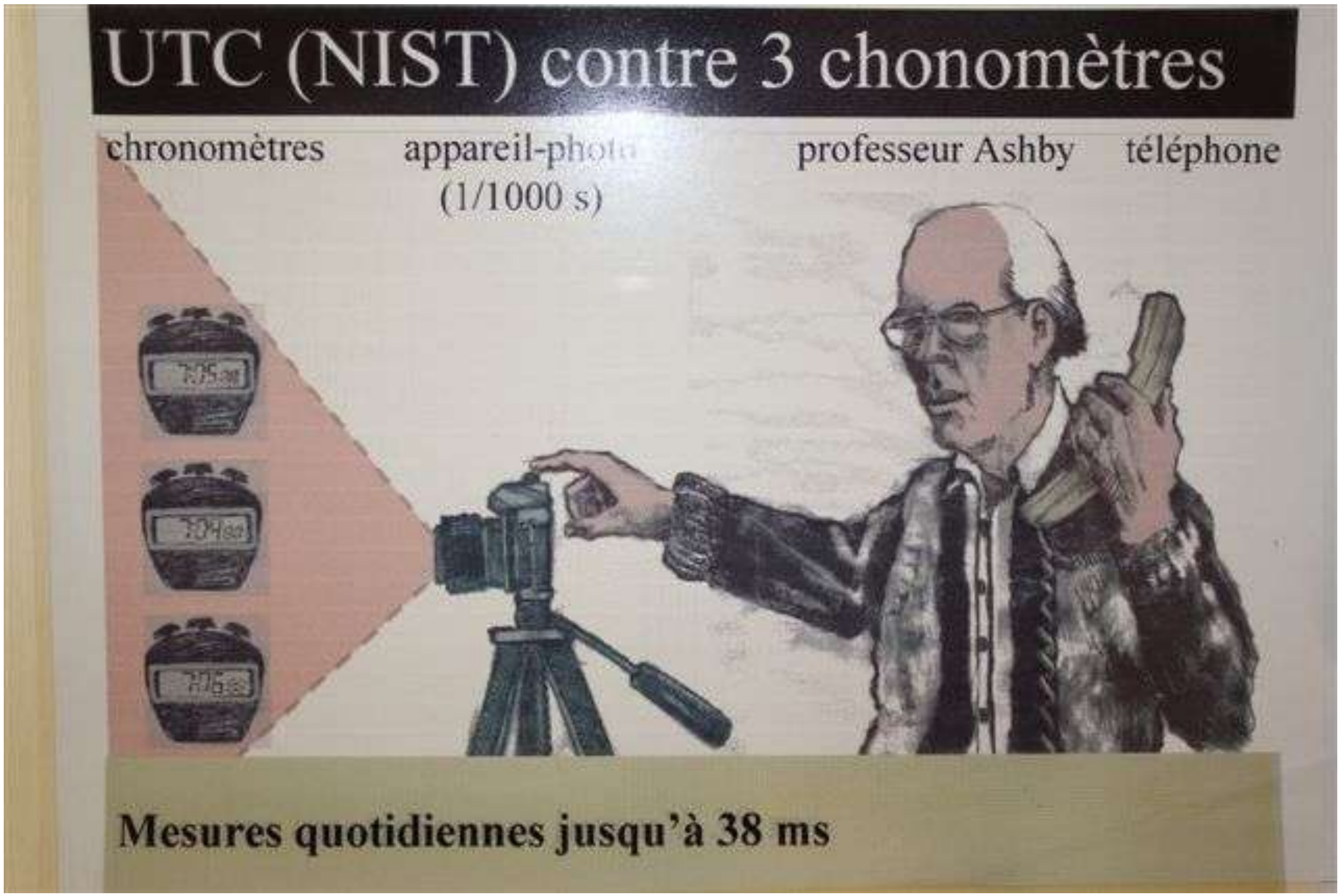
$$\therefore \ TIE\Big|_{drift} = \frac{\sqrt{6}}{2} \sigma_x(\tau) = 1.2 \ TDEV$$

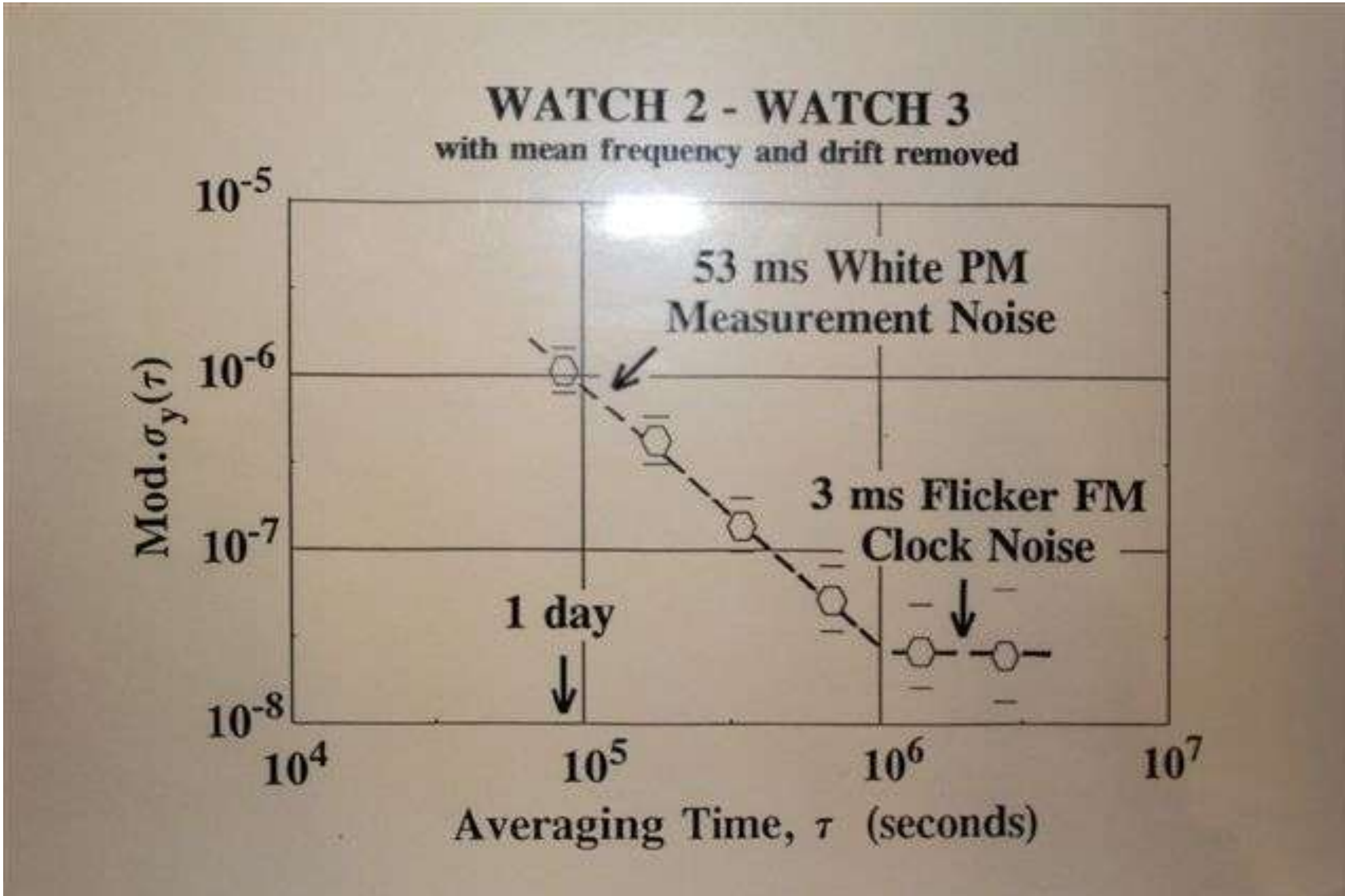


Time-keeping equation

$$x(t) = x_o + y_o t + \frac{1}{2} D t^2 + \varepsilon(t)$$

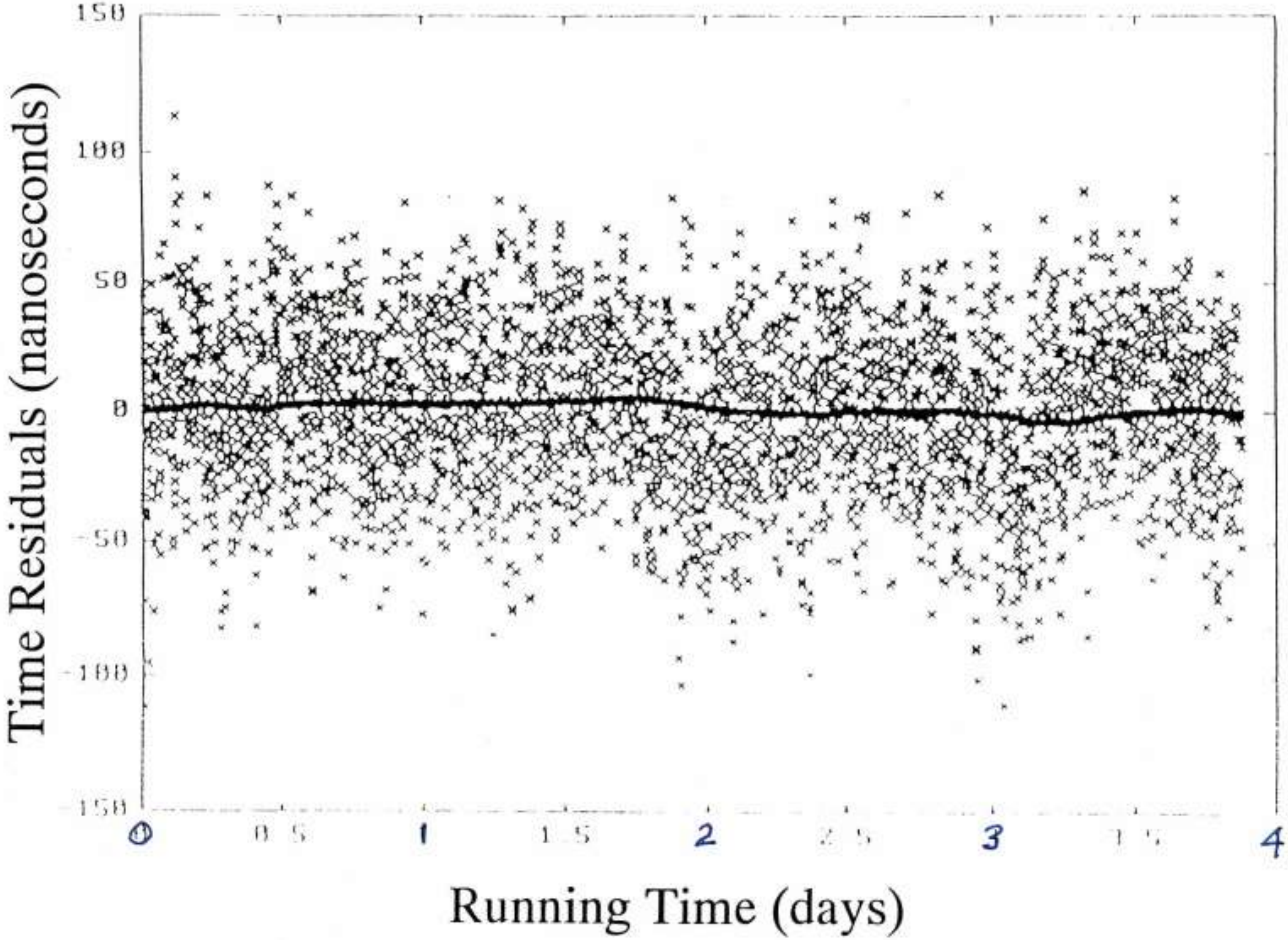
Prof. Ashby and 3 watches

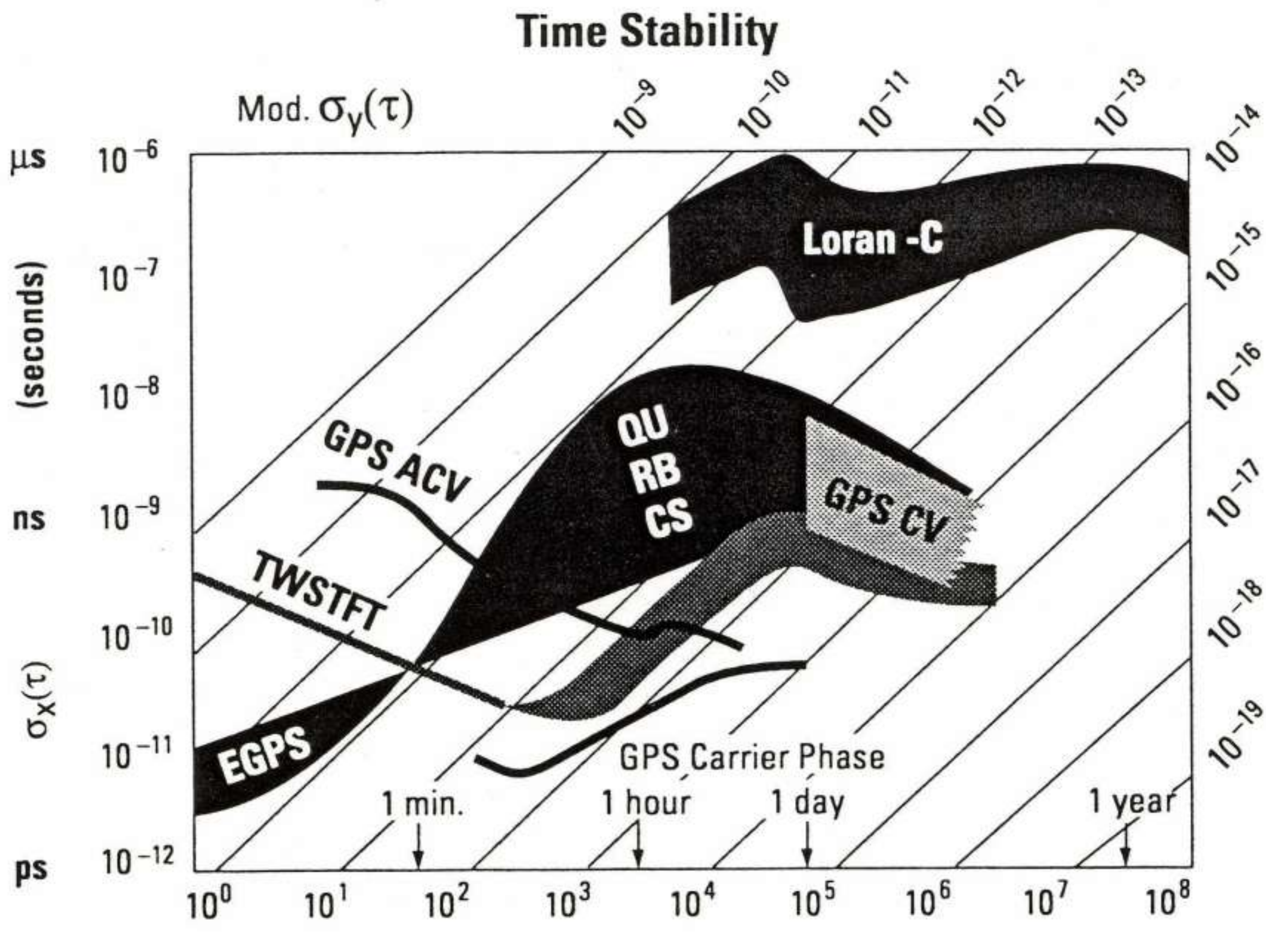




WATCH 2 - WATCH 3
with mean frequency and drift removed
10^-5
10^-6
10^-7
10^-8
Mod.σy(τ)
53 ms White PM
Measurement Noise
3 ms Flicker FM
Clock Noise
1 day
10^4
10^5
10^6
10^7
Averaging Time, τ (seconds)

GPS (with SA) versus Cesium Ensemble (Filtered and Unfiltered)



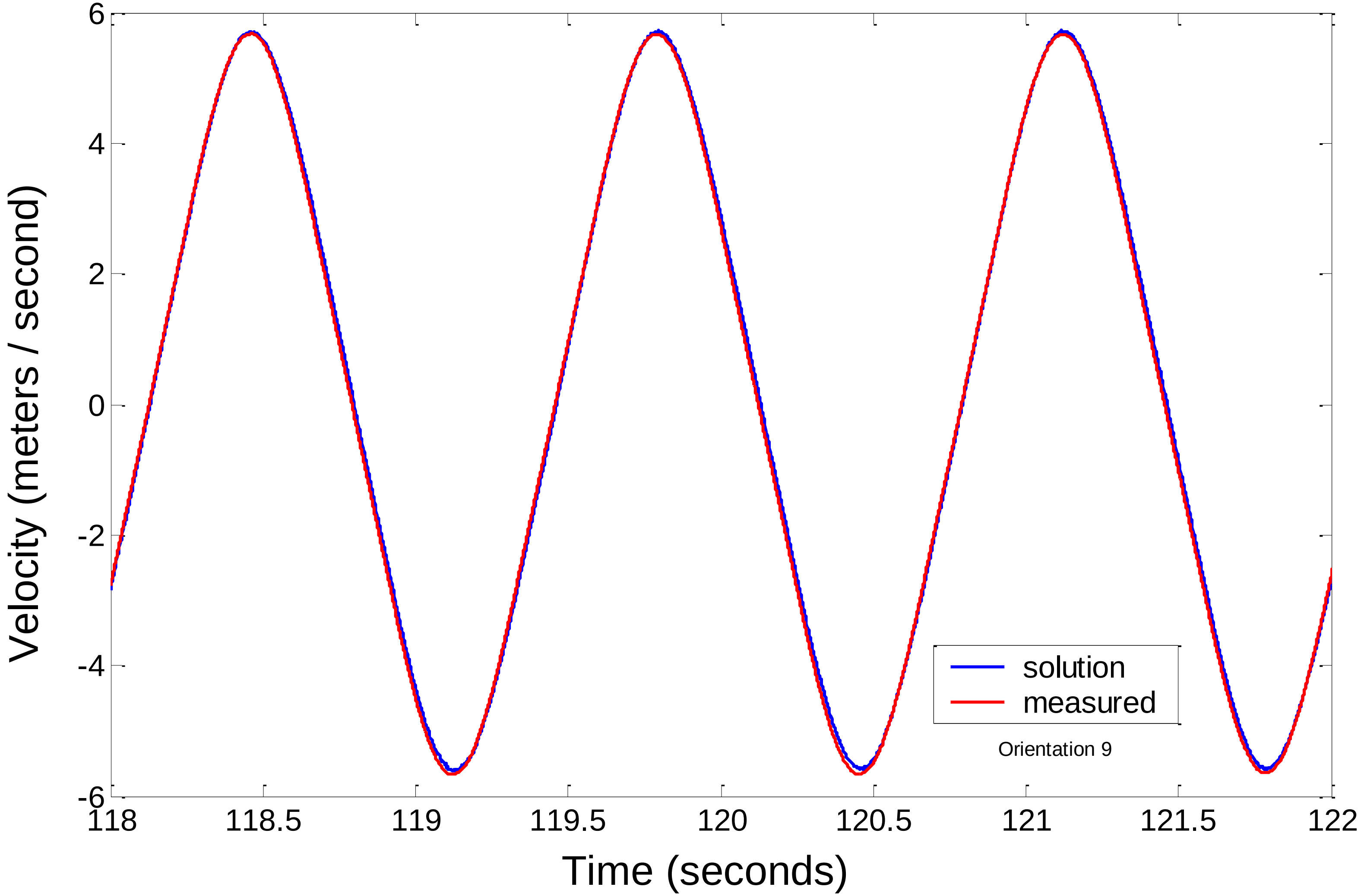


Time Stability
Mod. $\sigma_y(\tau)$
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
μs
ns
ps
$\sigma_x(\tau)$ (seconds)
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
Loran -C
QU
RB
CS
GPS ACV
GPS CV
TWSTFT
EGPS
GPS Carrier Phase
1 min.
1 hour
1 day
1 year
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
Sample Time, τ (seconds)

Exciting New T&F results With Quartz-crystal Oscillators

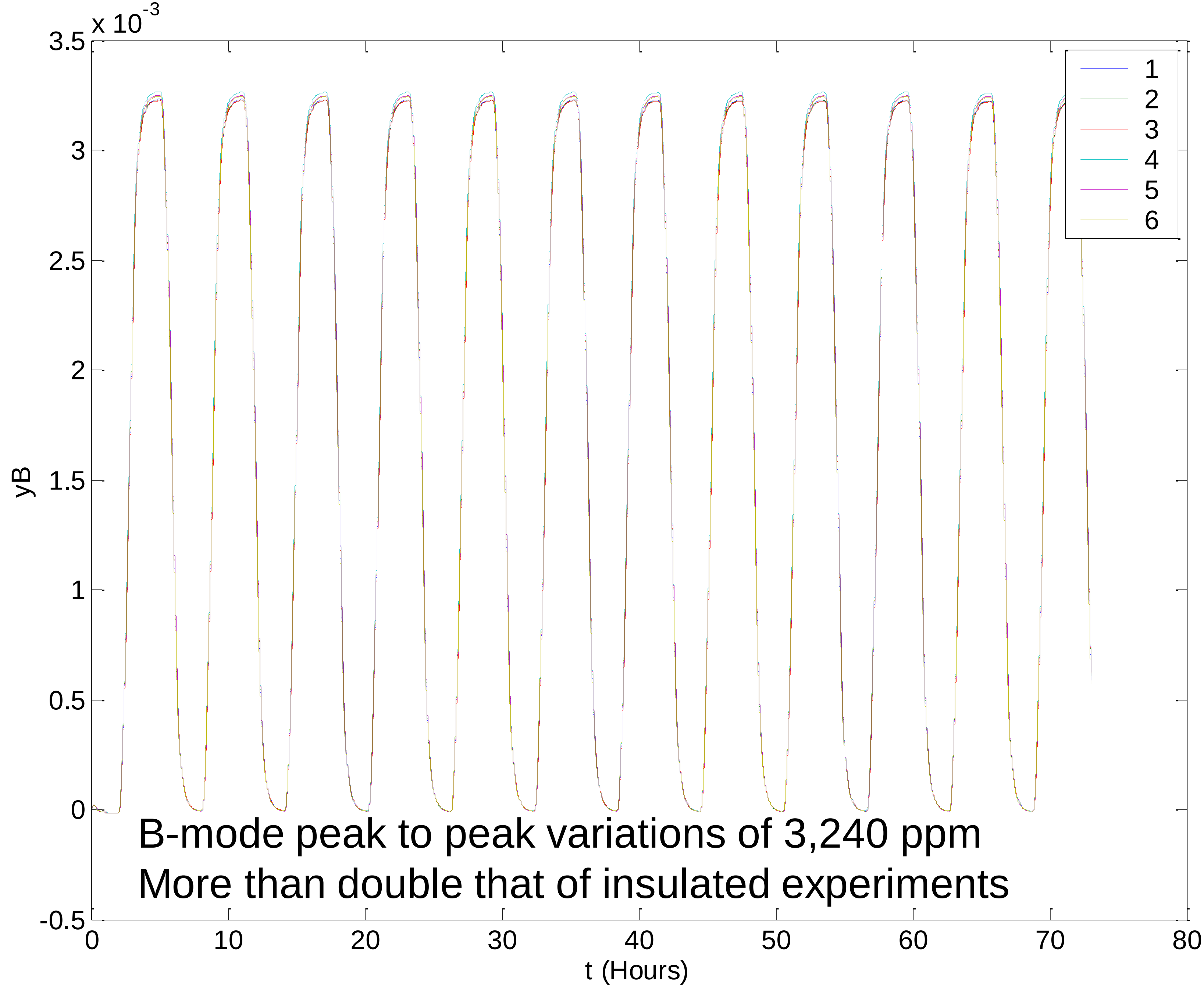
- Using EQUATE technology
- Ensemble of Quartz-crystal oscillators Adapting To the Environment
- Up to 5 g's of acceleration
- -40 to +85 deg. C temperature

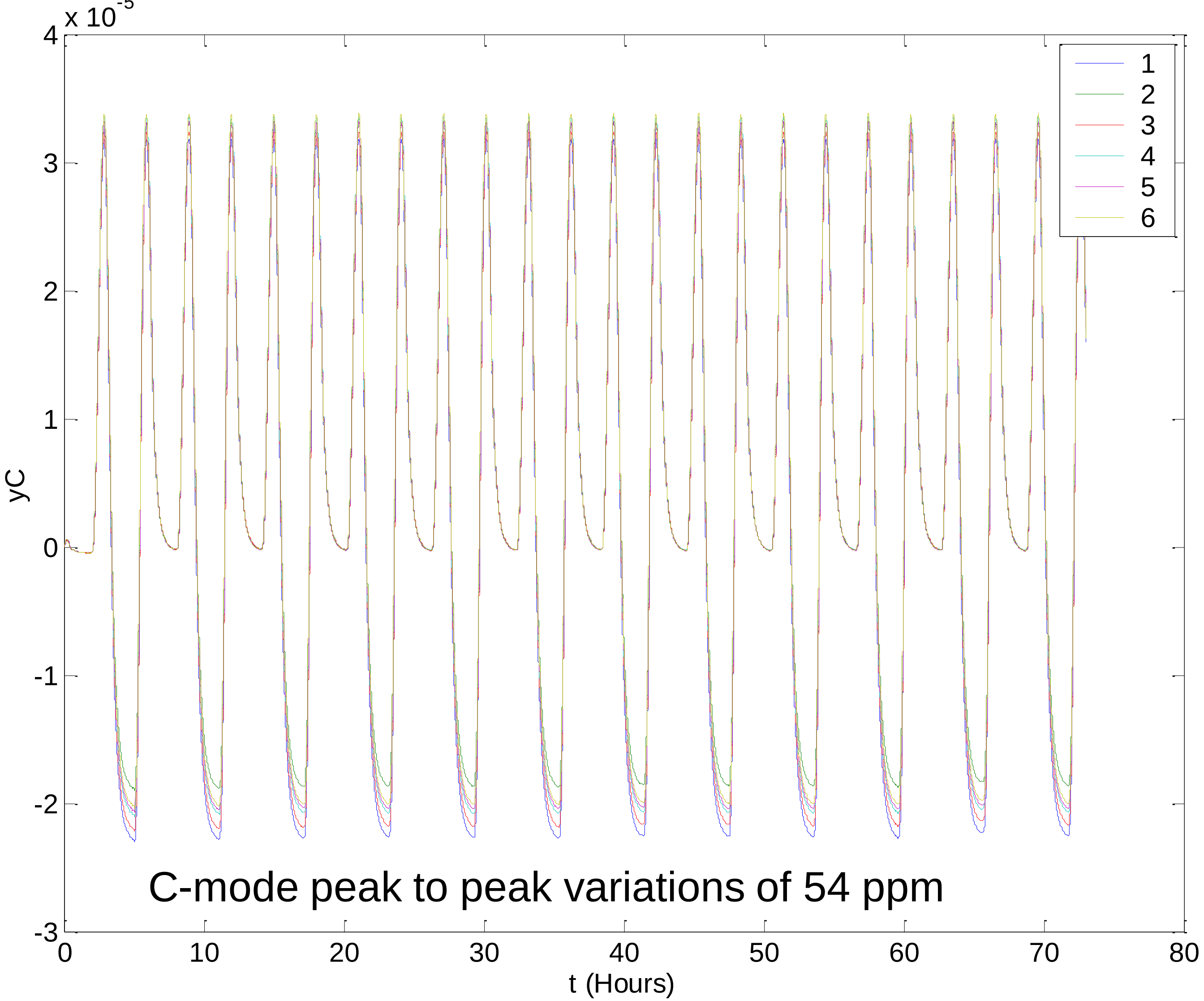

49°
1105

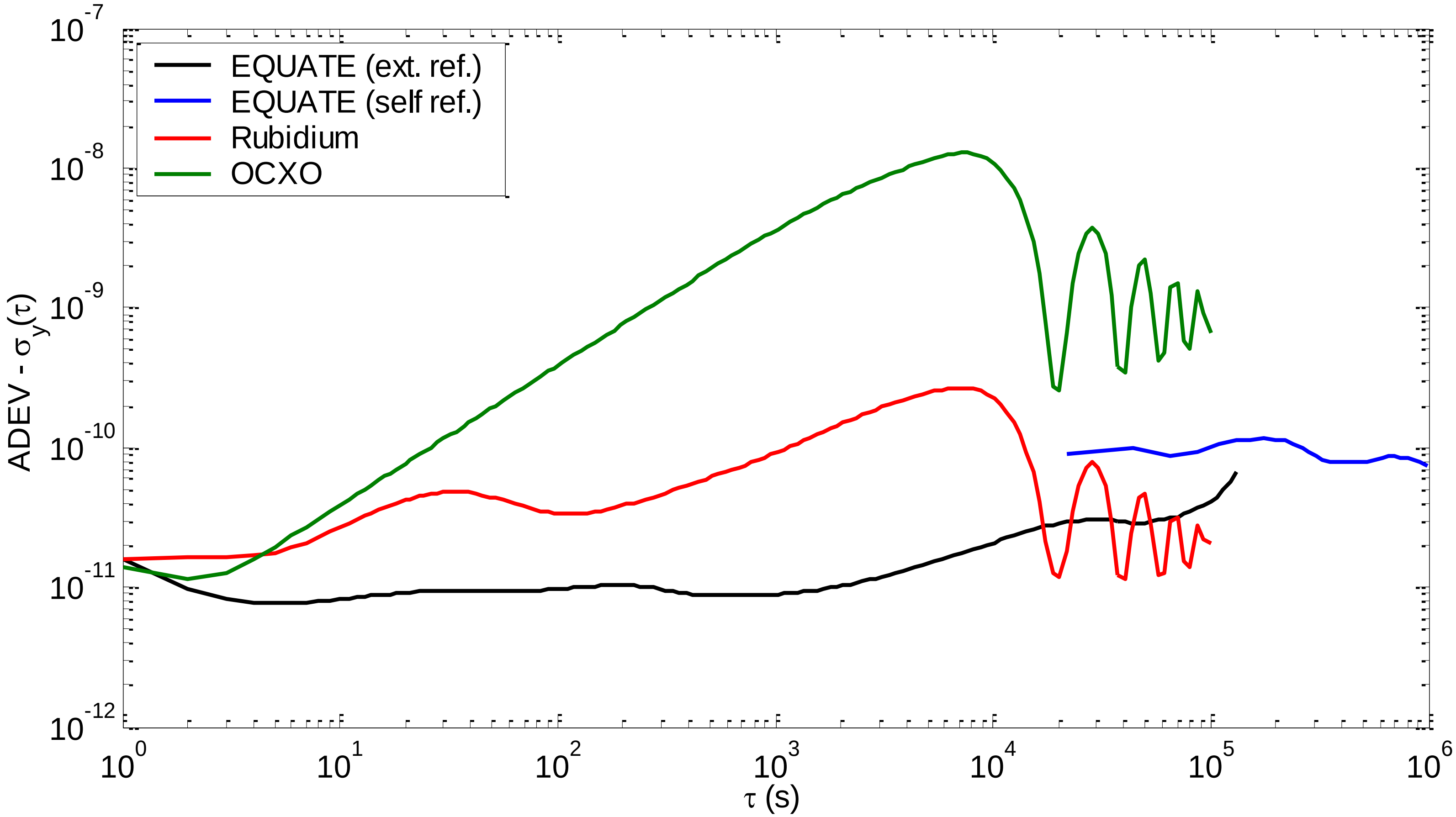


EQUATE velocity measurements

OVEN to test temperature independence of EQUATE timing package


x 10^-3
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
yB
0
10
20
30
40
50
60
70
80
t (Hours)
1
2
3
4
5
6
B-mode peak to peak variations of 3,240 ppm
More than double that of insulated experiments

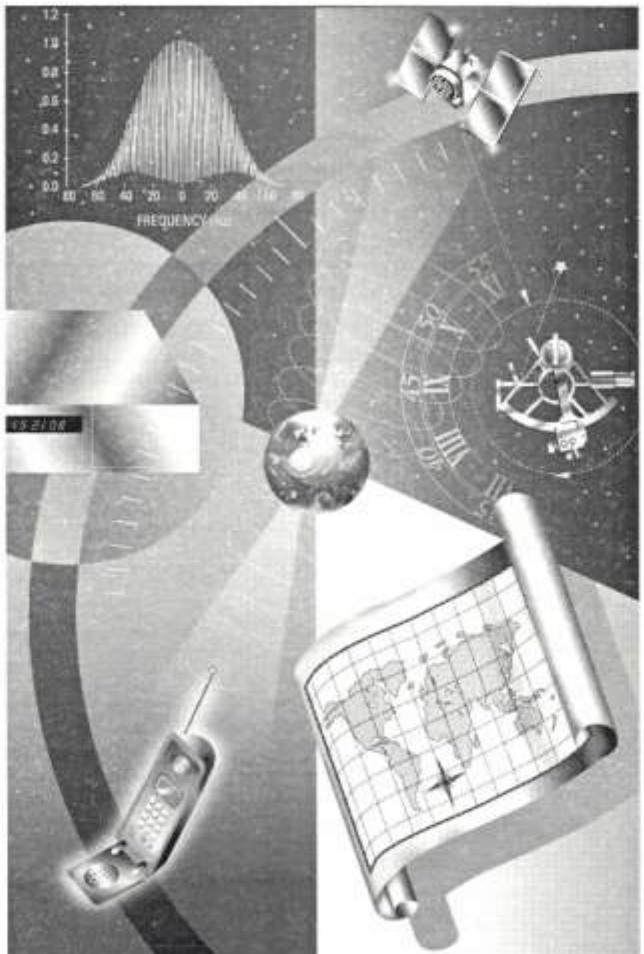

x 10^-5
4
3
2
1
0
-1
-2
-3
yC
0
10
20
30
40
50
60
70
80
t (Hours)
1
2
3
4
5
6
C-mode peak to peak variations of 54 ppm



Frequency Stability plot with wide temperature modulation with about six hour periods: for ovenless EQUATE (-40º to +85º C), for a commercial rubidium, and an OCXO (-20º to +65º C)

The Science of Timekeeping

Application Note 1289



- UTC: Official World Time
- GPS: A Time Distribution Utility
- Time: An Historical and Future Perspective
- From Laboratory to Practical Use
- Broad Applications Across Society

Time

Involves

Measurements

Extraordinaire