Time Error Estimation When UTC Time Traceability is Lost

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Introduction

- **Introduction**
  - Increasingly, precision time has become the critical parameter in synchronizing networks with migration first to LTE-TDD and now to 5G
  - The principal source of UTC time is GNSS, which, while exceptionally robust, is vulnerable to interference, jamming, spoofing and solar flares, and in rare cases system errors
  - Technologies such as the ePRTC (enhanced Primary Reference Time Clock), which couples a UTC source with an autonomous atomic clock, have been developed to address this
  - Whether the backup clock is cesium as it generally would be for the ePRTC, or quartz or rubidium, as it generally would be for the PRTC, understanding the time error when UTC traceability is lost, is important
- **Time Error Estimator Calculations**
  - **Random walk** (from frequency jump statistics)
  - **Holdover estimator** (from how well an oscillator conforms to predicted offset and drift)

The Clock Equation

Applicable to oscillators (e.g., Quartz, Rubidium, Cesium, H-Maser)

$$\phi = \phi_0 + \underset{\substack{\downarrow \\ \text{Offset}}}{\omega_0} t + \tfrac{1}{2}\underset{\substack{\downarrow \\ \text{Drift}}}{A}t^2 + \int_0^t E(T,P,\underset{\substack{\downarrow \\ \text{Environmental}}}{M},H,G)\,d\tau + \underset{\substack{\downarrow \\ \text{Random}}}{\varepsilon(t)}$$

Where:

ϕ_0 represents the starting synchronization error (initial phase offset)

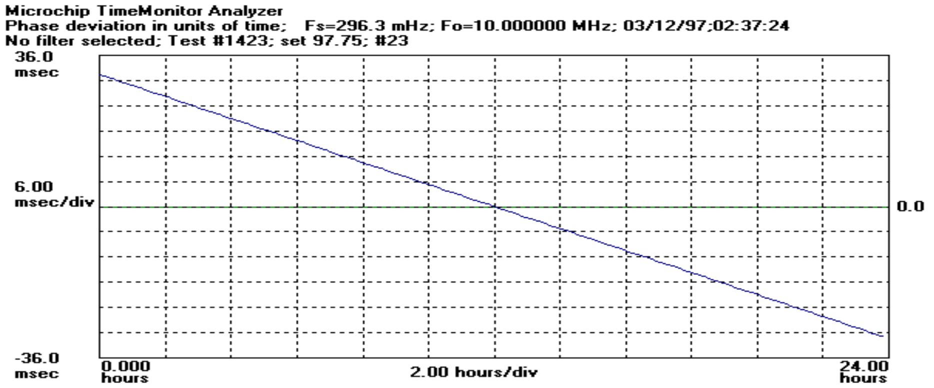
ω_0 represents the starting syntonization error (initial frequency offset)

A represents a linear aging term (but may be a function of time)

The integral represents environmental effects and is usually a function of time, and $\varepsilon(t)$ represents system noise and frequency jumps. The environmental parameters: T: temperature, P: pressure, M: magnetism, H: humidity, and G: generalized acceleration effects (gravity, vibration)

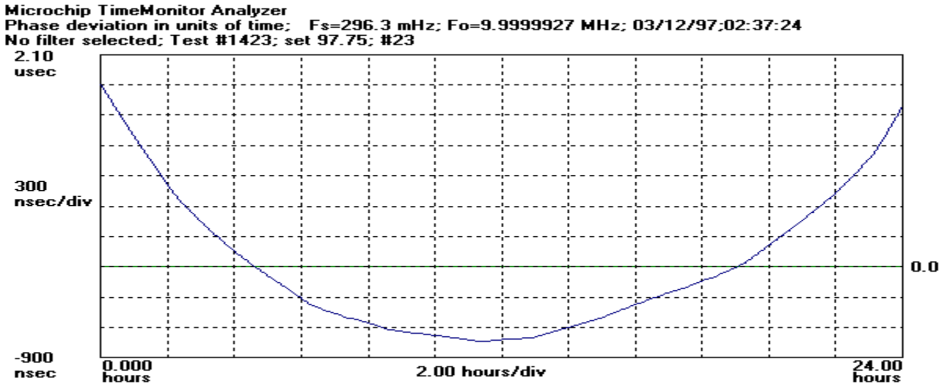
Oscillators: Offset and Drift Dominate

Relationship between phase and frequency: $\omega = \frac{d\phi}{dt}$



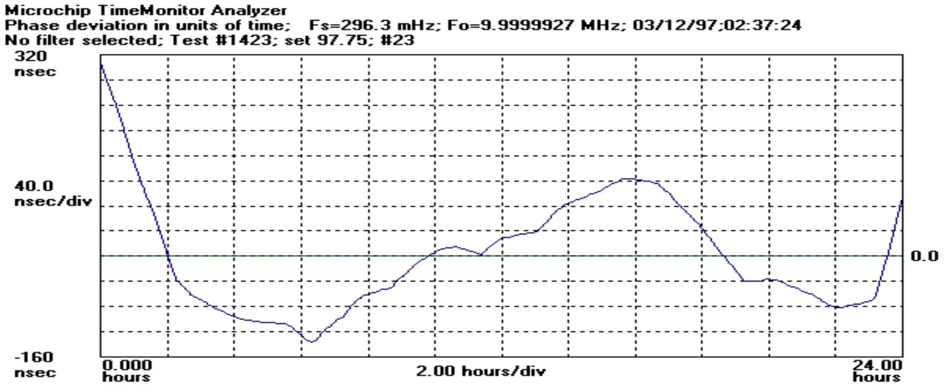
70 ms p-p

Original oscillator phase measurement (0.7ppm frequency offset, constant phase slope)



2.5 µs p-p

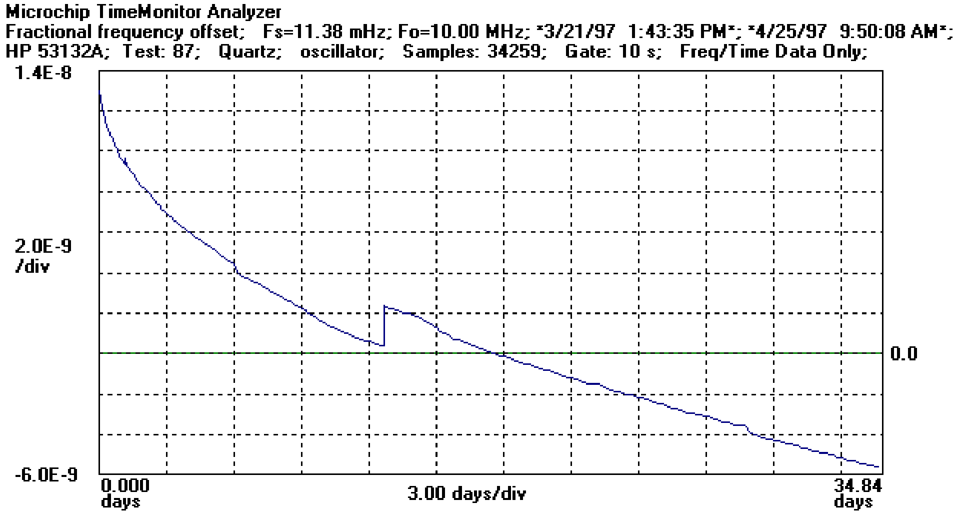
Frequency offset removed (quadratic shape shows linear frequency drift of 0.2 ppb/day; given phase/frequency relationship, quadratic in phase means linear in frequency)



440 ns p-p

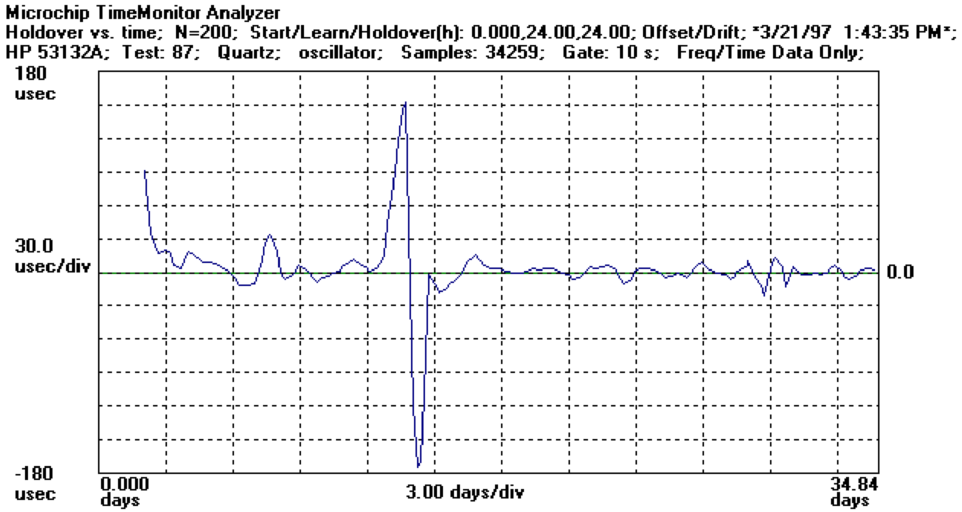
Frequency drift removed (shows residual phase movement)

Oscillator Frequency Jump: Effect on Holdover



Microchip TimeMonitor Analyzer
Fractional frequency offset; Fs=11.38 mHz; Fo=10.00 MHz; *3/21/97 1:43:35 PM*; *4/25/97 9:50:08 AM*;
HP 53132A; Test: 87; Quartz; oscillator; Samples: 34259; Gate: 10 s; Freq/Time Data Only;

Oscillator measurement with frequency jump at 12 days



Microchip TimeMonitor Analyzer
Holdover vs. time; N=200; Start/Learn/Holdover(h): 0.000,24.00,24.00; Offset/Drift; *3/21/97 1:43:35 PM*;
HP 53132A; Test: 87; Quartz; oscillator; Samples: 34259; Gate: 10 s; Freq/Time Data Only;

> 150 µs rather than 1 to 10 µs

The stochastic part is a very important component for determining performance!

The “random walk” and “holdover estimation” focus attention on this

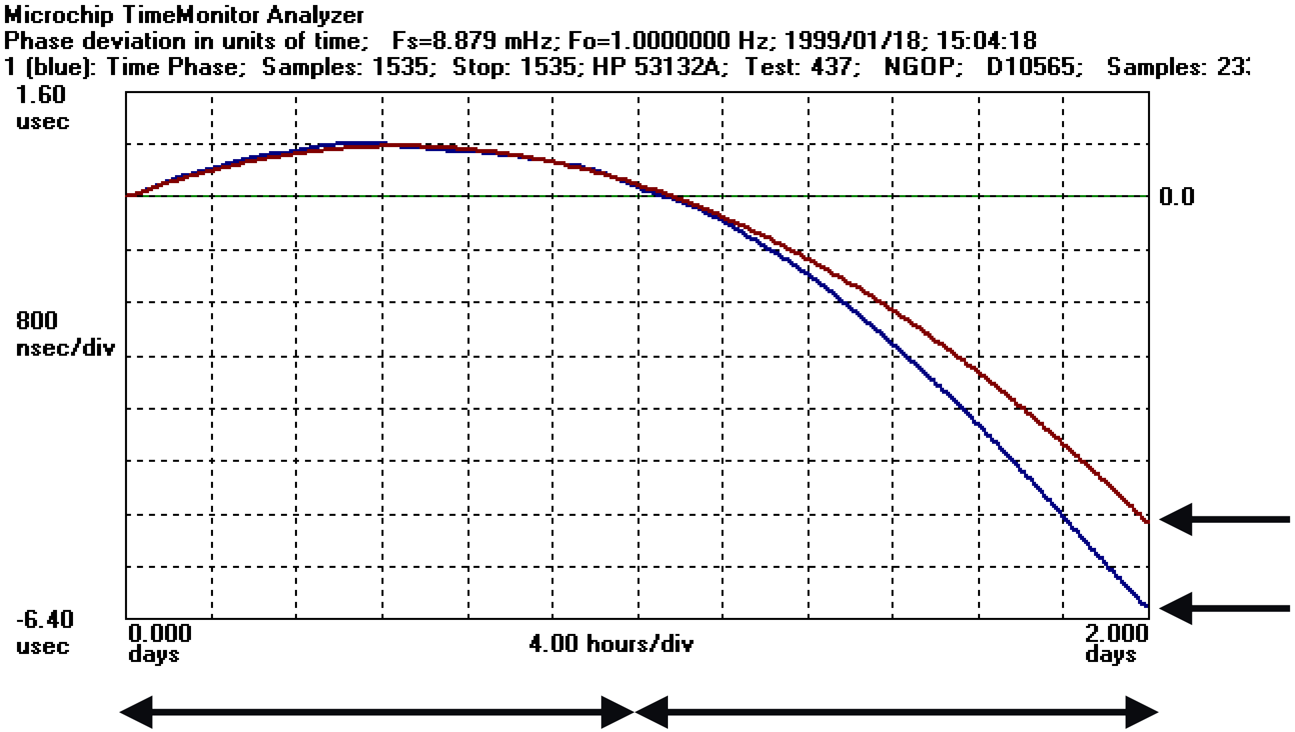
Random Walk

The difference between two adjacent frequency measurements is considered a “jump.” All jumps are stored in a histogram in accordance with their magnitudes. “Random Walk” is computed based on the statistics of jumps as the probable rms phase-walk. Data during warmup may be excluded.

$$\sigma_{rwZ}{}^{2} = \frac{Z^{2}}{3} \cdot \sigma_{jZ}{}^{2} = \frac{Z^{2} \cdot R}{6 \cdot \sum_{b=1}^{M} N_{b}} \sum_{b=1}^{M} N_{b} \left(\frac{h_{b}{}^{2} + h_{b} l_{b} + l_{b}{}^{2}}{3} \right) \sec$$

Random Walk calculation where Z is the random walk tau in seconds, T is tau in seconds, R is the ratio Z/T, M is the number of histogram bins, Nb is the number of frequency jumps in a particular bin b, l is the lower bin boundary, and h is the upper bin boundary.

Holdover Estimator



Microchip TimeMonitor Analyzer
Phase deviation in units of time; Fs=8.879 mHz; Fo=1.0000000 Hz; 1999/01/18; 15:04:18
1 (blue): Time Phase; Samples: 1535; Stop: 1535; HP 53132A; Test: 437; NGOP; D10565; Samples: 23:

Predicted time after 24 hours

Actual time after 24 hours

24-hour learning period, fit curve (in red) so it matches data (in blue)

24-hour holdover period

Through curve fit, determine these three coefficients from the clock equation:

$$\phi_0 + \omega_0 t + \tfrac{1}{2} A t^2$$

Holdover time error is difference between “predicted” and “actual”: 1.3 µs

Holdover Estimator

Quartz, Rubidium: $\phi_0 + \omega_0 t + \frac{1}{2} A t^2$

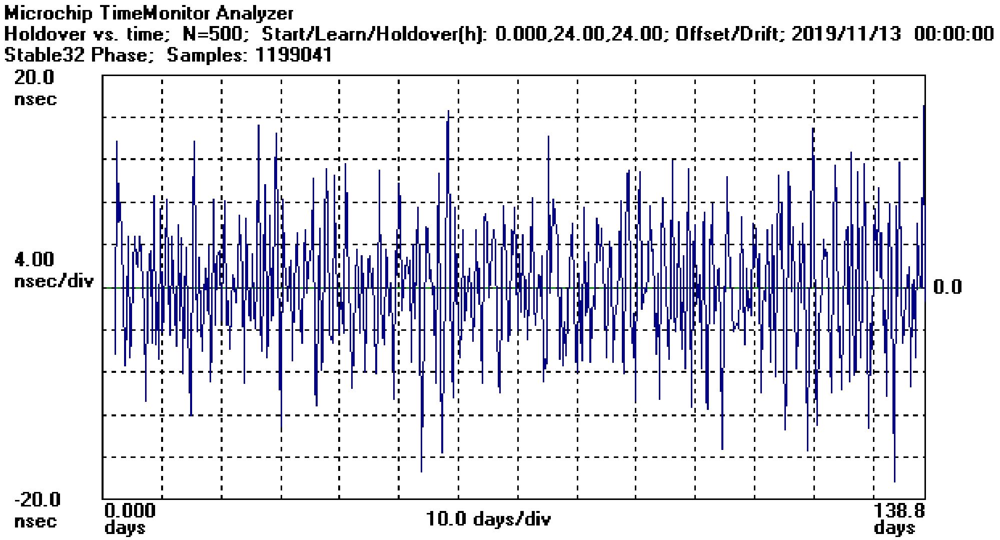
Fit to parabola

Cesium: $\phi_0 + \omega_0 t$

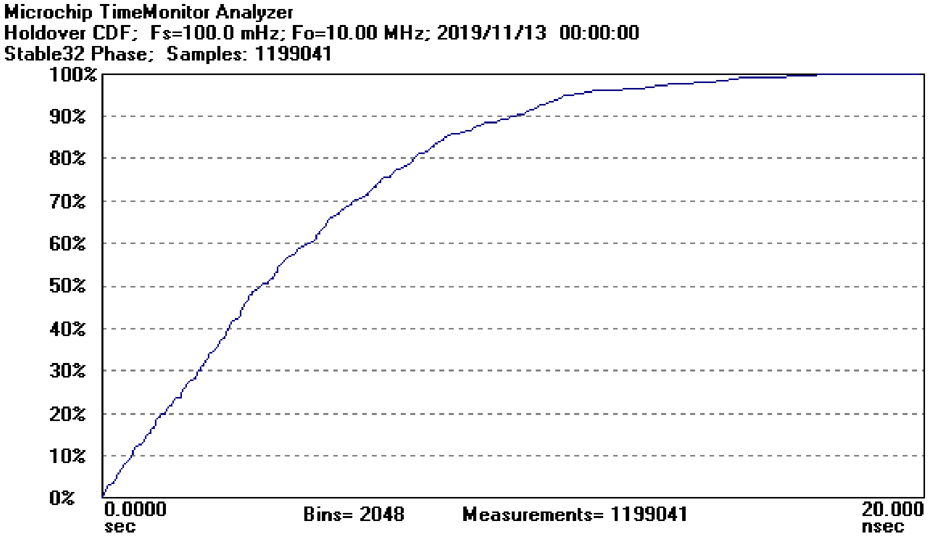
Fit to line

- "Holdover" computes frequency offset and drift during a specified learning period by fitting a 2nd order equation, and then uses that prediction of future drift for the specified holdover period. The predicted value is then compared to the actual value to compute a single holdover point; if the prediction is perfect then the holdover value is zero. Note for a primary reference such as a cesium clock, a 1st order equation fit is used as offset dominates.
- Then the starting point is moved forward in time and the whole procedure is repeated to compute another point. At the end, the statistics are calculated on the magnitude of the set of points, and a result determined, say the one-sigma or two-sigma point on the distribution.
- For a concrete example, consider an oscillator measured for three days with a 24-hour learning period and a 24-hour holdover period. If the starting point is advanced one hour starting with time zero, there will be 25 holdover points calculated to make use of the entire 3-day data set.

Holdover Estimator



Plot of holdover estimator results

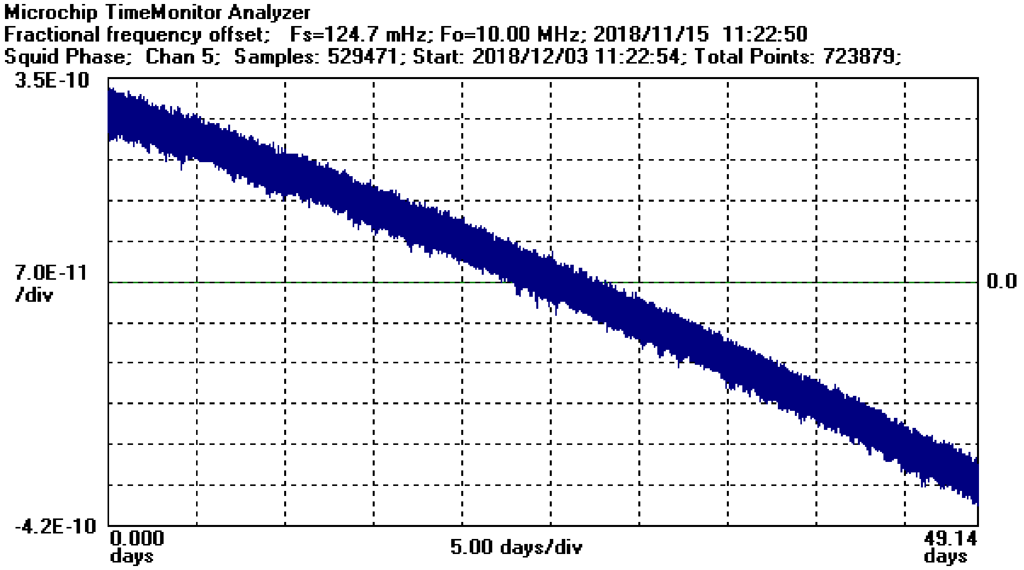


Cumulative distribution function of magnitude of the results

1 sigma: 4.41 ns

2 sigma: 10.35 ns

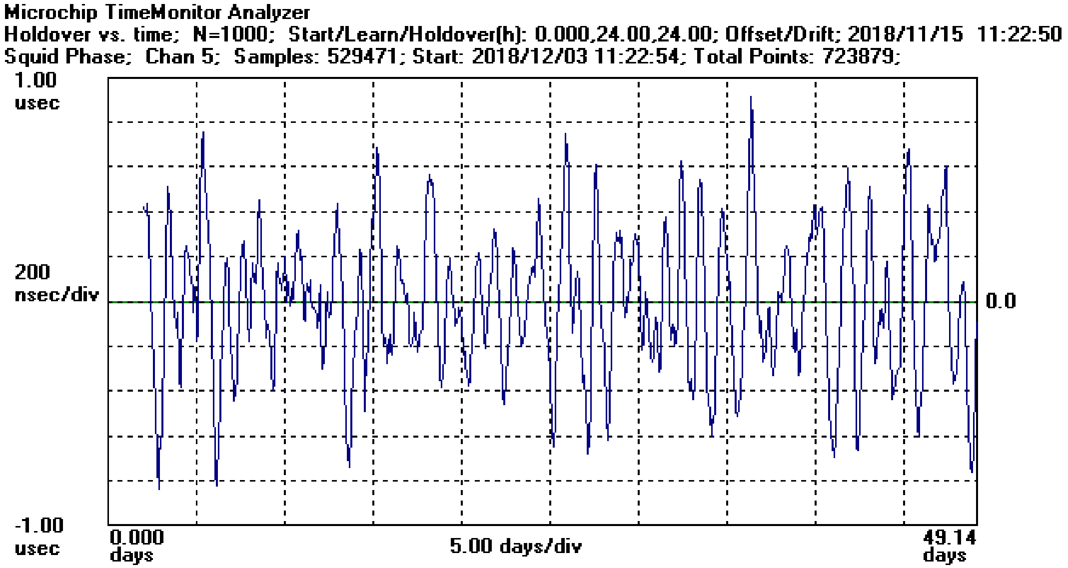
Time Error Estimator Example: Quartz



Microchip TimeMonitor Analyzer
Fractional frequency offset;   Fs=124.7 mHz; Fo=10.00 MHz; 2018/11/15 11:22:50
Squid Phase; Chan 5; Samples: 529471; Start: 2018/12/03 11:22:54; Total Points: 723879;

Raw oscillator measurement

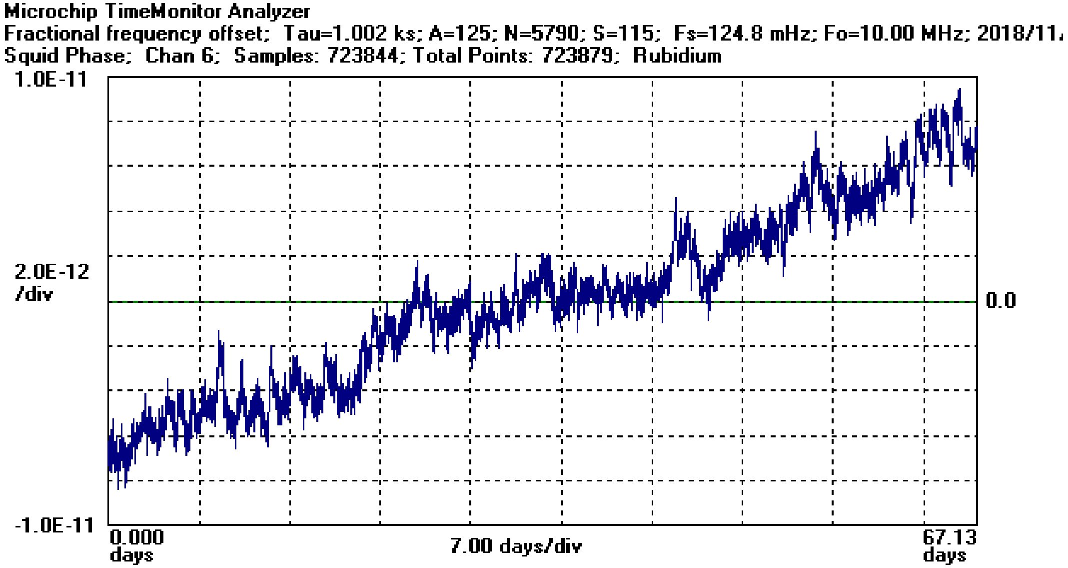
Random walk: 638 ns

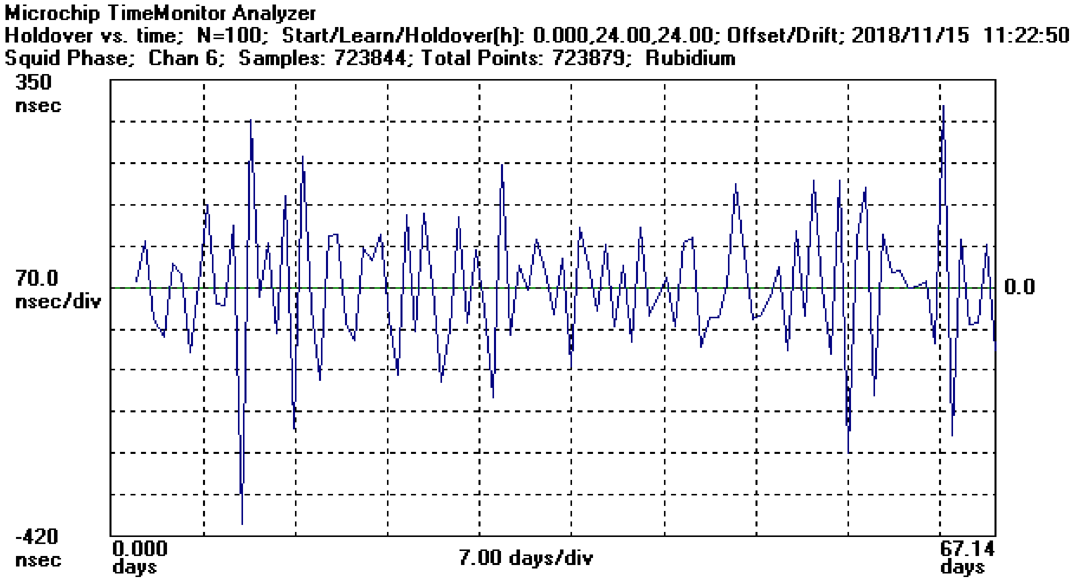


Microchip TimeMonitor Analyzer
Holdover vs. time; N=1000; Start/Learn/Holdover(h): 0.000,24.00,24.00; Offset/Drift; 2018/11/15 11:22:50
Squid Phase; Chan 5; Samples: 529471; Start: 2018/12/03 11:22:54; Total Points: 723879;

Holdover estimator: 620 ns

Time Error Estimator Example: Rubidium



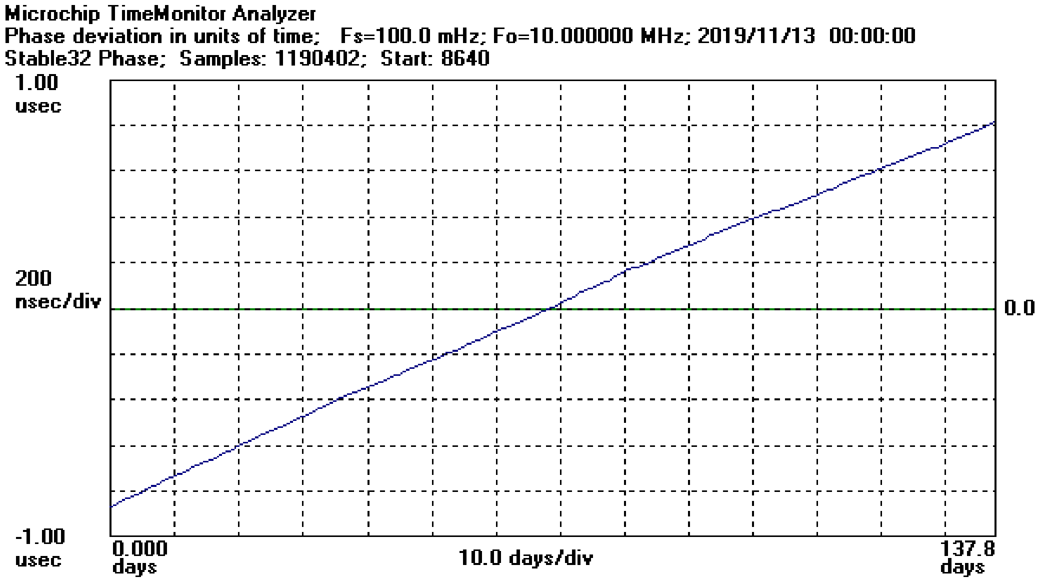


Raw oscillator measurement

Random walk: 201 ns

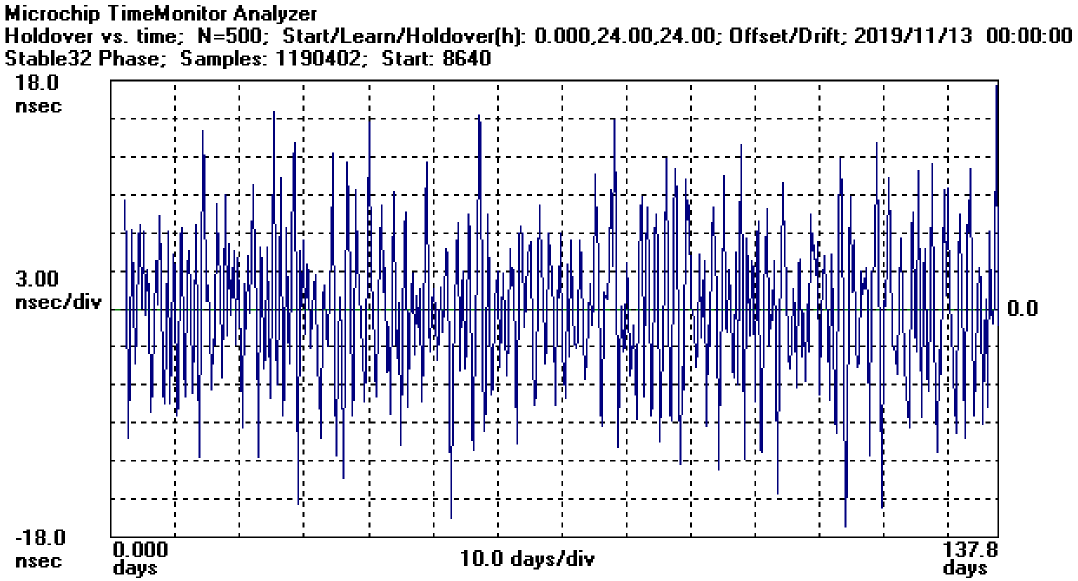
Holdover estimator: 188 ns

Time Error Estimator Example: Cesium



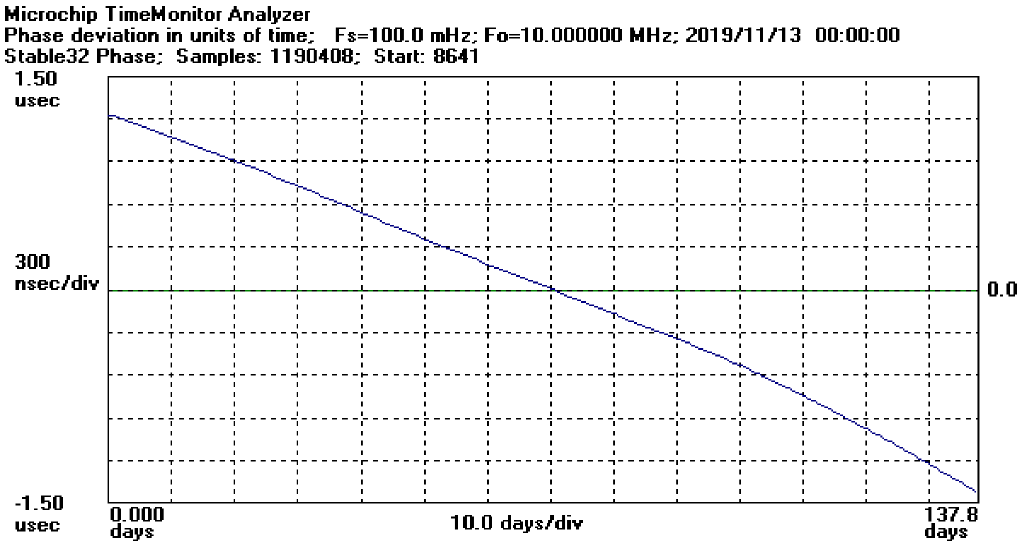
Raw oscillator measurement

Random walk: 14 ns



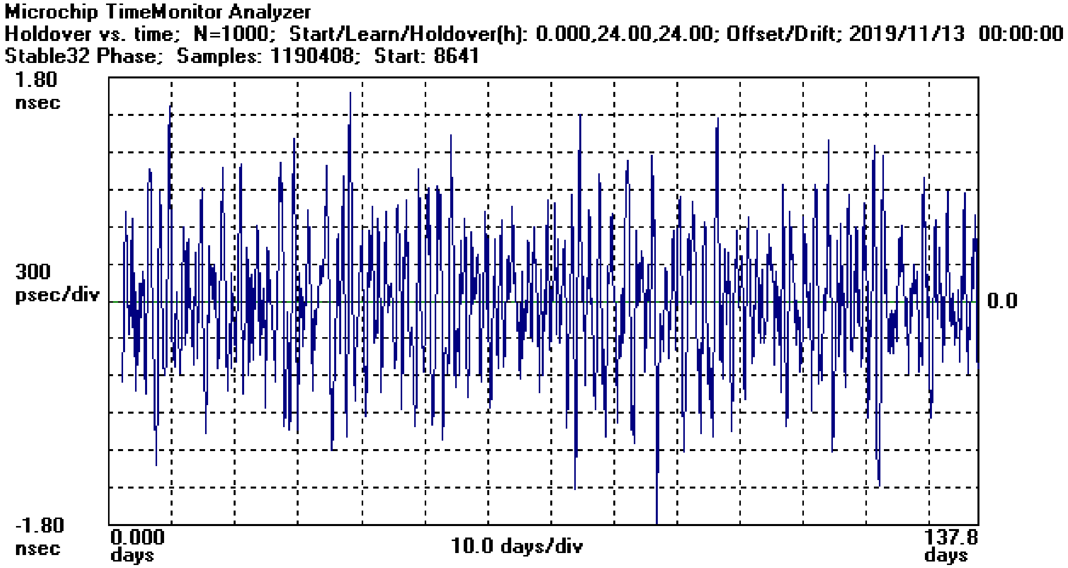
Holdover estimator: 14 ns

Time Error Estimator Example: H-Maser


Microchip TimeMonitor Analyzer
Phase deviation in units of time; Fs=100.0 mHz; Fo=10.000000 MHz; 2019/11/13 00:00:00
Stable32 Phase; Samples: 1190408; Start: 8641

Raw oscillator measurement

Random walk: 1.07 ns


Microchip TimeMonitor Analyzer
Holdover vs. time; N=1000; Start/Learn/Holdover(h): 0.000,24.00,24.00; Offset/Drift; 2019/11/13 00:00:00
Stable32 Phase; Samples: 1190408; Start: 8641

Holdover estimator: 818 ps

Time Error Estimator: 10 Quartz Oscillators

File count: 10		
Start: 8640		
ADEV(1000s); MDEV(7200s); TDEV(10000s)		
TUNC Start/Learn/Holdover(h): 0;24;24		
Random Walk FTau(seconds)/RWTau(days): 1000;1		
	TUNC(ns)	RW(ns)
Mean	1375	1291
Median	1414	1198
Stddev	242	223
Min	1003	1060
Max	1800	1677
1	1568	1216
2	1582	1114
3	1265	1069
4	1399	1677
5	1048	1179
6	1428	1070
7	1003	1563
8	1502	1447
9	1157	1515
10	1800	1060

TUNC (“time uncertainty”) is the same thing as the holdover estimator.

Note consistency between oscillators. More oscillators would be good to improve the statistics.

For these quartz oscillators, TUNC and random walk match reasonably well, so either or both could reasonably be employed as a predictor of holding time error

Time Error Estimator: Cesium and H-Maser

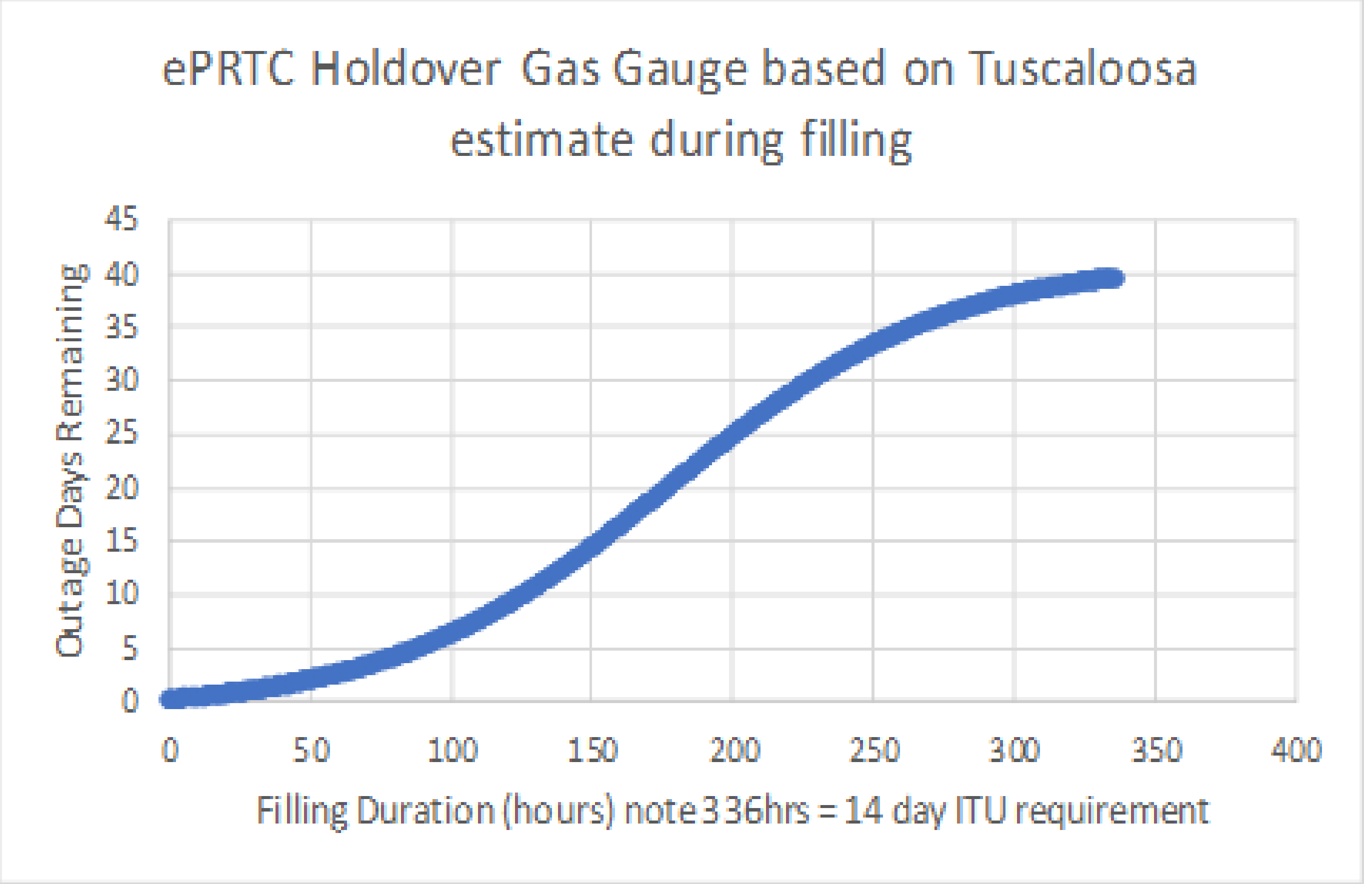
Random Walk FTau: 86400s			
Random Walk RWTau: 5 days			
TUNC in ns (0/120h/120h)			
	RW (ns)	TUNC(ns)	Days
Cesium1	23	40	182
Cesium2	13	24	138
Maser	2	4	138

TUNC (“time uncertainty”) is the same thing as the holdover estimator

Cesium learning period fit to first order equation as offset dominates

Random walk tau and learning/holdover period extended for these atomic clocks

Time Error Estimation Calculators: Application



"Filling" refers to the "learning period"

From this graph, for example, it can be seen that after 14 days of learning, ±100 ns can be held for an estimated 40 days, based on studying the autonomous cesium clock while UTC time via GNSS is available

Summary

- Holding time in a network is increasingly important given such technologies as TDD and 5G

- For determining predictive oscillator metrics, we start with the clock equation, which applies to all types of oscillators, from quartz to rubidium to cesium to hydrogen maser

- The stochastic term in the clock equation is critical: this is the part we can't predict or completely eliminate by design

- Two time error prediction calculations, "random walk" and "holdover estimation," are two different approaches for understanding the stochastic term

- When a system has access to traceable UTC time, the oscillator can be studied, helping to predict how well time can be held if UTC traceability goes away

Thank you

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